MA 224 - Quiz 8.5 Practice Problems

1. Suppose $f(x, y) = xy$, $x = e^{5t}$, and $y = e^{-t+1}$. Find $\frac{df}{dt}$ at $t = 1$.

2. A cylindrical can has a diameter of 10 meters and a height of 4 meters. If the cylinder has an insulating layer covering it that is .1 meters thick, use calculus to estimate the volume of the insulating layer.

   (Hint: think of the cylinder without the insulation as your original cylinder, and think of adding the insulation as just changing the dimensions of your original cylinder)

   (Second hint: if the insulation is .1 m thick, what does that mean about how much you are changing the radius and height of the cylinder by?)

3. page 625 #37, Using x skilled workers and y unskilled workers, a manufacturer can produce $Q(x, y) = 60x^{1/3}y^{2/3}$ units per day. Currently the manufacturer employs 10 skilled workers and 40 unskilled workers and is planning to hire 1 additional skilled worker. Use calculus to estimate the corresponding change that the manufacturer should make in the level of unskilled labor so that the total output will remain the same.

Answers

Keep in mind there are other CORRECT ways to express these answers, but the answers I provide are likely to be the way the answers would appear on an exam.

1. Use the chain rule: $df/dt = f_x \cdot \frac{dx}{dt} + f_y \cdot \frac{dy}{dt} = f_x x'(t) + f_y y'(t)$.

The answer is $4e^5$

2. Using the incremental approximation formula we get that the answer is $\Delta V = 9\pi$.

The formula for volume is $V = \pi \cdot r^2 \cdot h = \pi r^2 h$, and the formula for incremental approximation is $\Delta V = V_r \cdot \Delta r + V_h \cdot \Delta h$.

Since the insulation is .1 thick all the way around, that means that radius is increased by $\Delta r = .1$, but the height is increased by $\Delta h = .2$.

3. First of all, the phrase "use calculus to estimate" should indicate to you that you need to use the incremental approximation formula, $\Delta Q = Q_x \Delta x + Q_y \Delta y$

Second of all, if the output remains the same, that means it doesn't change! If Q doesn't change, then $\Delta Q = 0$.

The answer is that the level of unskilled workers should be decreased by 2 workers, i.e. $\Delta y = -2$.