MA 224 - Quiz 8

SOLUTIONS

NOTE: My solution for a given problem is not necessarily the only correct way to do that problem; I accept any method of solving the problems that follows the instructions and uses material taught in this course. I give a lot of detail to make sure everything is clear—I don't expect you to give quite this much detail.

1. (7 pts) Find all critical points, BUT DO NOT CLASSIFY THEM

$$f(x,y) = (x+y)e^{1-x-y^2}$$

First we find f_x and f_y, and then we set them both equal to 0.

$$\begin{aligned} f_x &= e^{1-x-y^2} + (x+y)e^{1-x-y^2}(-1) \\ &= e^{1-x-y^2}\left(1+(x+y)(-1)\right) && \text{factor} \\ &= e^{1-x-y^2}\left(1-x-y\right) \end{aligned}$$

$$\begin{aligned} f_y &= e^{1-x-y^2} + (x+y)e^{1-x-y^2}(-2y) \\ &= e^{1-x-y^2}\left(1+(x+y)(-2y)\right) && \text{factor} \\ &= e^{1-x-y^2}\left(1-2xy-2y^2\right) \end{aligned}$$

Now we set each equal to 0:

$$\begin{aligned} f_x :& e^{1-x-y^2}\left(1-x-y\right) = 0 \\ f_y :& e^{1-x-y^2}\left(1-2xy-2y^2\right) = 0 \end{aligned}$$

since e^{1-x-y^2} is non-zero, we can divide by it:

$$\begin{aligned} f_x :& 1-x-y = 0 \\ f_y :& -2xy-2y^2 = 0 \end{aligned}$$

If we could get rid of the x in the f_y equation, we'd be able to solve for y! So we use the f_x equation to express x in terms of y: $1-x-y=0$ becomes $x = 1-y$. Plugging this into x in the f_y equation gives

$$\begin{aligned} f_y :& 1-2(1-y)y-2y^2 = 0 \\ & 1-2y+2y^2-2y^2 = 0 && \text{distribute} \\ & 1-2y = 0 \end{aligned}$$

This tells us $y = \frac{1}{2}$. And from our equation above, $(x = 1-y)$, we know that $x = 1-y = 1-\frac{1}{2} = \frac{1}{2}$. Hence, the only critical point is $(\frac{1}{2}, \frac{1}{2})$.

2. (3 pts) If $g(s,t)$ has the property that $g_{ss} = -1$, $g_{tt} = -2$ and $g_{st} = -1$, what can you say about any critical point?

Using the formula for the discriminant in the Second Partials test, $D = g_{ss}g_{tt} - (g_{st})^2$, we get that $D = (-1)(-2) - (-1)^2 = 2 - 1 = 1 > 0$, i.e. $D > 0$ for *any* critical point (since there are no x's and y's in the formula, the formula doesn't depend on what the critical points are).

Since $D > 0$, that means *any* critical point must be a max or a min.

To check whether a critical is a max or a min, look at g_{ss}. Since $g_{ss} = -1 < 0$, that means *any* critical point must be a max. We know this even without knowing what the critical point is!