MA 224 - Quiz 9

SOLUTIONS

NOTE: My solution for a given problem is not necessarily the only correct way to do that problem; I accept any method of solving the problems that follows the instructions and uses material taught in this course. I give a lot of detail to make sure everything is clear—I don't expect you to give quite this much detail.

1. I am making a box with a square bottom, and I am using several different materials. The bottom is made out of a material that is twice as expensive as the materials used for the sides and the top. I want the box to have volume = 1500. What dimensions should I use to make the box as cheap as possible?

1.1 (3 points) State what quantity you are trying to optimize, then write down a function that gives that quantity.

We are trying to minimize the cost of the box (make it as cheap as possible). Since the cost of the box is just the cost of the materials we need to make the box, the cost depends on the amount of material we're using—since the walls of the box are thin, we just estimate how much material we're using by using the surface area of all the sides. We only use *one* of the sides (don't count the surface area of both sides of each wall).

Let x be the length of the sides, and let y be the height of the box. The amount of area of one side is then xy. But there are 4 sides, so $4xy$ total area of the sides. The top has area x^2. The bottom has area x^2 as well, but it costs twice as much, so it counts as $2x^2$. Adding this together we get $C(x, y) = 4xy + 2x^2 + x^2 = 4xy + 3x^2$ is the function we're trying to minimize.

1.2 (2 points) What is the constraint equation for this problem?

Since the volume has to be 1500, and volume is length times width times height, the constraint is $x^2y = 1500$. That means $g(x, y) = x^2y$

1.3 (5 points) Use the method of Lagrange Multipliers to solve the problem. If you do not test your critical point(s), you will get no credit for this question.

First set up the Lagrange equations:

$$\begin{aligned} 4y + 6x &= \lambda 2xy \\ 4x &= \lambda x^2 \\ x^2y &= 1500 \end{aligned}$$

If $x \neq 0$ then we can divide equation 2 by x to get $4 = \lambda x$. We can plug this into equation 1:

$$\begin{aligned} 4y + 6x &= \lambda 2xy = 2y \cdot (\lambda x) \\ &= 2y \cdot 4 \\ 6x &= 8y - 4y \\ \frac{6}{4}x &= y \end{aligned}$$

Now we can plug this into the constraint equation:

$$\begin{aligned} x^2 y &= 1500 \\ x^2(6x/4) &= 1500 \\ x^3 &= 1000 \\ x &= 10 \end{aligned}$$

And since $y = 3x/2$, we get $y = 15$. So $(10, 15)$ is the answer.

Note, if you use different letters for your variables, you might get $y = 10$, $x = 15$—that's totally correct. What's important is that the width is 10 and the height is 15.

But we have to CHECK this answer. We do this by picking **any other point that satisfies the consraint equation**, and then plugging both into the original function and comparing:

$x = 1$ and $y = 1500$ satisfies the constraint equation.

$C(1, 1500) = 4 * 1 * 1500 + 3 * (1)^2 = 6003$.

$C(10, 15) = 4 * 10 * 15 + 3 * (10)^2 = 900$, so clearly (10,15) is where the minimum cost occurs.