10 Find a basis for the submodule of $\mathbb{Z}^{(3)}$ generated by $(1, 0, -1), (2, -3, 1), (0, 3, 1), (3, 1, 5)$.

10 Over the polynomial ring $\mathbb{Q}[x]$ find a base for the submodule of $\mathbb{Q}[x]^{(3)}$ generated by $(2x-1, x, x^2+3)$, (x, x, x^2), and $(x+1, 2x, 2x^2 - 3)$.

10 Find a basis for the $\mathbb{Z}$-submodule of $\mathbb{Z}^{(3)}$ consisting of the set of all (x, y, z) satisyfing $x + 2y + 3z = 0$ and $x + 4y + 9z = 0$.

10 Put this matrix into normal form:
$$\begin{bmatrix} 6 & 2 & 3 & 0 \\ 2 & 3 & -4 & 1 \\ -3 & 3 & 1 & 2 \\ -1 & 2 & -3 & 5 \end{bmatrix}$$

10 Determine the structure of $M = \mathbb{Z}^{(3)}/K$ where K is generated by $f_1 = (2, 1, -3)$ and $f_2 = (1, -1, 2)$.

10 Let $D = \mathbb{R}[X]$ and assume that M is a direct sum of cyclic D-modules whose order ideals (annihilating ideals) are the ideals generated by the polynomials $(x - 1)^3, (x^2 + 1)^2, (x - 1)(x^2 + 1)^4, (x + 2)(x^2 + 1)^2$. Determine the elementary divisors of M and the invariant factors of M.

10 If N is a direct summand of M (i.e. $M = N \oplus K$, show N is pure in M.

10 If N is a pure submodule of M and $\text{ann}(x + N) = (d)$, prove that w can be chosen in $x + N$ such that $w + N = x + N$ and $\text{ann}(w) = (d)$.

10 If N is a pure submodule of a finitely generated module M over a PID D, prove that N is a direct summand of M. You may assume that a finitely generated module of a PID is a direct product of cyclic modules.