12) Classify up to similarity all matrices $A \in \mathbb{C}^{3\times 3}$ such that $A^3 = I$.

12) For $n \in \mathbb{N} \geq 2$ let N be an $n \times n$ matrix over a field F such that $N^n = 0$ but $N^{n-1} \neq 0$. Show that N has no square root, i.e. there is no $A \in F^{n\times n}$ such that $A^2 = N$.

12) If N is a nilpotent matrix in $\mathbb{C}^{3\times 3}$ then prove that $A = I + \frac{1}{2}N - \frac{1}{8}N^2$ satisfies $A^2 = I + N$. Use the binomial expansion formula on $(1+x)^{1/2}$ to obtain a similar formula for a square root of $I + N$ for N a nilpotent matrix over $\mathbb{C}^{n\times n}$.

12) Use 15 to prove that for nonzero $c \in \mathbb{C}$ and nilpotent $N \in \mathbb{C}^{n\times n}$ we know $(cI + N)$ has a square root. Then use Jordan form to prove that non-singular matrices $\in \mathbb{C}^{n\times n}$ have square roots.

12) True or false: every matrix in $F[x]^{n\times n}$ is row-equivalent to an upper-triangular matrix?

12) Let T be a linear operator on the n-dimensional vectorspace V. Let A be the matrix representation of T in the ordered basis B. Show that T has a cyclic vector iff the determinants of the $(n-1)\times(n-1)$ submatrices of $xI - A$ are relatively prime.

12) Let A be an $n \times n$ matrix with entries in the field F and let $f_1, f_2, \cdots, f_n$ be the diagonal entries of the normal form of $xI - A$. For which matrices A is $f_1 \neq 1$?

12) Construct T with minimal polynomial $x^2(x-1)^2$ and characteristic polynomial $x^3(x-1)^4$. Describe the primary decomposition of V under T and find the projections on the primary components.

12) If N is a nilpotent linear operator on V, show that for any polynomial f the semi-simple part of $f(N)$ is a scalar multiple of I.

12) Let F be a subfield of the complex numbers, V a finite-dimensional vectorspace over F and T a semi-simple operator on V. If f is any polynomial over F, prove that $f(T)$ is semi-simple.

12) Let T be a linear operator on n-dimensional V over F a subfield of $\mathbb{C}$. Prove that T is semi-simple iff "if f is a polynomial over F and $f(T)$ is nilpotent, then $f(T) = 0$."