13) Let $V = \mathbb{C}^2$ with the standard inner product. Let T be the linear operator defined by $T\epsilon_1 = (1, -2), T\epsilon_2 = (i, -1)$. Let $\alpha = (x_1, x_2)$ and find $T^*\alpha$.

13) Let T be the linear operator on $\mathbb{C}^2$ defined by $T\epsilon_1 = (1 + i, 2)$ and $T\epsilon_2 = (i, i)$. Find the matrix of T^* in the standard ordered basis. Does T commute with T^*?

13) Show that the range of T^* is the orthogonal complement of $\operatorname{null} T$, i.e. show $R = R(T^*) = (\operatorname{null}(T))^\perp = N$.

13) Let V be a finite dimensional inner product space (fin dim IPS), and T a linear operator on V. If T is invertible, show that T^* is invertible and that $(T^*)^{-1} = (T^{-1})^*$.

13) Show that the product of 2 self-adjoint operators is self-adjoint iff the two operators commute.

13) Let V be a fin dim IPS over $\mathbb{C}$. Let E be a projection operator / an idempotent operator on V. Prove E is self-adjoint iff E is normal, i.e. $E = E^*$ iff $E^*E = EE^*$.

13) Let V be a fin dim IPS over $\mathbb{C}$. Let T be a linear operator on V. Show that T is self-adjoint iff $(Tx|x)$ is real for all $x \in V$.

13) For each matrix A, find a real orthogonal matrix P such that P^TAP is diagonal.

13) Is a complex symmetric matrix self-adjoint? Is it normal?

13) Give an example of a 2×2 matrix A such that A^2 is normal but A is not normal.

13) Prove that a real symmetric matrix has a real symetric cube root.

13) Prove that a normal and nilpotent operator is the zero operator.

13) If T is a normal operator, prove that characteristic vectors for T which are associated with distinct characteristic values are orthogonal.

13) Let T be a normal operator on V a fin dim IPS over $\mathbb{C}$. Prove that there is a polynomial $f \in \mathbb{C}[x]$ such that $T^* = f(T)$.

13) If two normal operators commute, prove that their product is normal.