

1) Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.

1) Give an example of a system of two linear equations in two unknowns that has no solutions.

1) For what values of y_1, y_2, y_3 does the system of equations represented by the matrix equation $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$, abbreviated $AX = Y$ have a solution?

1) Find 2 nonzero matrices $A \in F^{n \times n}$ such that $A^2 = 0$.

1) A possible set of elementary matrices are exactly those corresponding to any set of elementary row operations to row-reduce A into I .

1) Prove that for a matrix $C \in F^{2 \times 2}$, $C = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$ there exist matrices $A, B \in F^{2 \times 2}$ such that $C = AB - BA$ iff $w + z = 0$ i.e. $z = -w$.

1) Consider $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$. For which X does there exist a scalar c satisfying $AX = cX$?

1) If A is $m \times n$ and B is $n \times m$ with $n < m$, prove that the $n \times n$ square matrix AB cannot be invertible.