4) If A and B are $n \times n$ matrices over the field F show that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$. Then show that similar matrices have the same trace.

4) If A and B are complex matrices, show that $AB - BA = I$ is impossible.

4) Let $V = \mathbb{R}^{2\times 2}$ and let $B = \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$. Let W be the subspace of V consisting of all A such that $AB = 0$. Let f be a linear functional on V which is in the annihilator of W and such that $f(I) = 0$ and $f(C) = 3$ where $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. Find $f(B)$.

4) Let V be a finite dimensional vectorspace over a field F and let W be a subspace of V. If f is a linear functional on W prove that there exists a linear functional g on V such that $g(x) = f(x)$ for all $x \in W$.

4) Let F be a field of characteristic 0 and let V be a finite dimensional vectorspace over F. If $a_1, \cdots, a_m$ are finitely many vectors in V, each not the zero vector, prove that there is a linear functional f on V such that $f(a_i) \neq 0$, for $i = 1, \cdots, m$.

4) Let V be the vectorspace of $n \times n$ matrices over a field F and let W_0 be the subspace of V spanned by matrices of the form $AB - BA$. Show that W_0 is exactly the space of matrices with trace 0.

4 Let S be a set, F a field, and $V(S, F)$ the space of all functions from S into F. Let W be any n-dimensional subspace of $V(S, F)$. Show that there exist n points $x_1, \cdots, x_n$ in S and n functions $f_1, \cdots, f_n$ in W such that $f_i(x_j) = \delta_{ij}$.

4 Let F be a field a let f be the linear functional on F^2 defined by $f(x, y) = ax + by$. Let $g = T^t f$. Find g for
(a) $T(x, y) = (y, 0)$.
(b) $T(x, y) = (-y, x)$.
(c)$T(x, y) = (y, 0)$.

4) Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Let c be a scalar and suppose there is a non-zero vector $a \in V$ such that $Ta = ca$. Prove that there is a non-zero linear functional f on V such that $T^t f = cf$.

4) Let A be an $n \times m$ matrix over $\mathbb{R}$. Show that $A = 0$ iff $\mathrm{tr}\, AA^T = 0$.