5 Let T be the linear operator on R^3 defined by $T(x, y, z) = (x, z, -2y - z)$. Let f be the polynomial over R defined by $f = -x^3 + 2$. Find $f(T)$.

5 Let A be an $n \times n$ diagonal matrix over the field F. Let f be the polynomial over F defined by $f = (x - A_{11}) \cdots (x - A_{nn})$. What is the matrix $f(A)$?

5 For $a, b \in F$ a field and $a \neq 0$ show that $B = \{1, ax + b, (ax + b)^2, (ax + b)^3, \cdots\}$ is a basis for $F[X]$.

5 If F is a field and $h \in F[X]$ of degree ≥ 1 show that the mapping $f \mapsto f(h)$ is a one-to-one linear transformation of $F[X]$ into $F[X]$. Show that this transformation is an isomorphism iff $\deg h = 1$.

5 Use Lagrange Interpolation to find f such that $\deg f \leq 3$ satisfying $f(-1) = -6; f(0) = 2; f(1) = -2; f(2) = 6$.

5 Let L be a linear functional on $F[X]$ such that $L(fg) = L(f)L(g)$ for all $f, g \in F[X]$. Show that either $L = 0$ or there is a t in F such that $L(f) = f(t)$ for all $f\ inF[X]$.

5 If $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$, find the monic polynomial that generates the ideal of all polynomials $f \in F[X]$ such that $f(A) = 0$.

5 Assuming the fundamental theorem of algebra, prove that if f and g are polynomials over $\mathbb{C}$, then $\gcd(f, g) = 1$ iff f and g have no common root.

5 Let D be the differentiation operator. Assuming the fundamental theorem of algebra, show that for a polynomial f over $\mathbb{C}$ we have f has no repeated roots iff $\gcd(f, Df) = 1$.

5 In which of these cases is D a 3-linear function?
(a)$D(A) = A_{11} + A_{22} + A_{33}$ 2nd row are unchanged, but $sD(A) + D(A') = sA_{11} + sA_{22} + sA_{33} + A'_{11} + A_{22} + A_{33}$, which is clearly not the same for all matrices A.
(b)$D(A) = (A_{11})^2 + 3A_{11}A_{22}$
(c)$D(A) = A_{11}A_{12}A_{33}$
(d)$D(A) = A_{13}A_{22}A_{32} + 5A_{12}A_{22}A_{32}$
(e)$D(A) = 0$
(f)$D(A) = 1$

5 Let K be a subfield of $\mathbb{C}$ and $n \in \mathbb{N}$. Let $j_1, \cdots, j_n$ and $k_1, \cdots, k_n$ be positive integers not exceeding n (i.e. they can each represent a row number of an $n \times n$ matrix over K). Let $A \in K^{n \times n}$ and defined $D(A) = A(j_1, k_1) \cdots A(j_n, k_n)$. Prove that D is n-linear iff the integers $j_1, \cdots, j_n$ are distinct.

5 Let F be a field and D be a function on $F^{n \times n}$. Suppose $D(AB) = D(A)D(B)$ for all $A, B \in F^{n \times n}$. Show that either $D(A) = 0$ for all A, or $D(I) = 1$.