6) List explicitly the six permutations of degree 3, state which are odd and which are even, and use this to give the complete formula for the determinant of a 3×3 matrix.

6 Using the traditional notation for a permutation, let $\sigma = (1234)$ and $\tau = (132)$.
(a) Are σ and τ odd or even?
(b) Find $\sigma\tau$ and $\tau\sigma$.

6 Let $A \in F^{n\times n}$ be a triangular matrix. Prove its determinant is equal to the product of its diagonal entries.

6 Show that $\det(xI - A)$ is a 3rd degree monic polynomial.

6 Let T be a linear operator defined on F^n and define $D_T(a_1, \cdots, a_n) = \det(Ta_1, \cdots, Ta_n)$
(a) Show D_T is n-linear and alternating.
(b) If we let $c = \det(T\epsilon_1, \cdots, T\epsilon_n)$ then show for any n vectors $v_1, \cdots, v_n$ we have $\det(Tv_1, \cdots, Tv_n) = c\det(v_1, \cdots, v_n)$.
(c) If B is any ordered basis for F^n and A is the matrix of T in this ordered basis, show that $\det A = c$.
(d) What is a reasonable name for the scalar c

6 If A is $n \times n$ over $\mathbb{C}$ for odd n, and $A^T = -A$, i.e. A is skew symmetric, then show $\det A = 0$.

6 If A is orthogonal show $\det A = \pm 1$.

6 Let A be $n \times n$ over $\mathbb{C}$ and suppose that it is unitary, i.e. $AA^* = I_n$. Show that $|\det(A)| = 1$.

6 Let $A \in F^{n\times n}$. Prove that there are at most n distinct scalars c in F such that $\det(cI - A) = 0$.

6 Let A and B be $n \times n$ over F. Show that if A is invertible there are at most n scalars c such that $cA + B$ is not invertible.