7 Let T be the linear operator on $\mathbb{R}^4$ which is represented in the standard ordered basis by the matrix $M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \end{bmatrix}$ Under what conditions on a, b, c is T diagonalizable?

7 Let T be a linear operator on the n-dimensional vector space V, and suppose that T has n distinct characteristic values. Prove that T is diagonalizable.

7 Let A and $B \in F^{n \times n}$. Prove that if $(I - AB)$ is invertible then $(I - BA)$ is invertible with inverse $I + B(I - AB)^{-1}A$.

7 Prove for $A, B \in F^{n \times n}$ that AB and BA have precisely the same characteristic values in F.

7 Suppose that $A \in \mathbb{R}^{2 \times 2}$ is symmetric. Prove that A is similar over $\mathbb{R}$ to a diagonal matrix.

7 Let $N \in \mathbb{C}^{2 \times 2}$ such that $N^2 = 0$. Prove that either $N = 0$ or N is similar over $\mathbb{C}$ to $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

7 If $A \in \mathbb{C}^{2 \times 2}$ show that A is similar over $\mathbb{C}$ to either a diagonal matrix or a matrix of the form $\begin{bmatrix} s & 0 \\ 1 & s \end{bmatrix}$.

7 Let $V = F^{n \times n}$. Let A be a fixed element of V. Let T be the linear operator on V given by left multiplication by the matrix A. Is it true that A and T have the same characteristic values?

7 Let T be a linear operator on V with $\dim V = n$ such that $T^k = 0$ for some $k \in \mathbb{N}$. Prove that $T^n = 0$.

7 Find a 3×3 matrix for which the minimal polynomial is x^2.

7 Let V be the space of polynomials $\in \mathbb{R}[x]$ that have degree at most n and let D be the differentiation operator.

7 Let P be an operator on $\mathbb{R}^2$ such that $P(x, y) = (x, 0)$. Show that P is linear. What is its minimal polynomial?

7 Let V be the vectorspace of $n \times n$ matrices over F a field. Fix $A \in V$ and let T be the linear operator on V defined by $T(M) = AM$ for all $M \in V$. Show that the minimal polynomial for T is the minimal polynomial for A.

7 Let $A, B \in F^{n \times n}$. Do AB and BA have the same characteristic polynomials? The same minimal polynomials?

7 Let T be the linear operator on $\mathbb{R}^2$ having matrix representation in the standard ordered basis $A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$. (a) Prove that the only subspaces of R^2 invariant under T are R^2 and 0. (b) If U is the linear operator on C^2, the matrix of which is given by A, show that U has a 1-dimensional invariant subspace.

7 Let W be an invariant subspace for T. Prove that the minimal polynomial for the restriction operator T_W divides the minimal polynomial for T, without referring to matrices.

7 Let c be a characteristic value of T and let W be the space of the characteristic vectors associated with the characteristic value c. What is the restriction operator T_W?

7 Show that every matrix A such that $A^2 = A$ is similar to a diagonal matrix.

7 Let T be a linear operator on a finite dimensional vectorspace over $\mathbb{C}$. Show T is diagonalizable iff T is annihilated by a polynomial over $\mathbb{C}$ with distinct roots.

7 Let T be a linear operator on V a vectorspace over a field F. If every subspace of V is invariant under T, show that T is multiplication by a scalar.

7 Let A be a 3x3 matrix with real entries. Prove that if A is not similar over R to a triangular matrix, then A is similar over C to a diagonal matrix.

7 True or false: if the triangular matrix A is similar to a diagonal matrix, then A is already diagonal?

7 Let T be a linear operator on V an n-dimensional vectorspace over an algebraically closed field F. Let f be a polynomial over F. Show that c is a char val of $f(T)$ iff $c = f(t)$ for t a char val of T.

7 Let V be the space of $n \times n$ matrices over F. Let A be a fixed $n \times n$ matrix over F. Let T and U be linear operators on V defined by

$T(B) = AB$

$U(B) = AB - BA$.

(a) True or false: If A is diagonalizable over F then T is diagonalizable over F. (b) True or false: If A is diagonalizable, then U is diagonalizable.