8 Let F be a commuting family of matrices in $\mathbb{C}^{3\times 3}$. How many linearly independent matrices can F contain? What about the $n \times n$ case?

8 Let T be a linear operator on an n-dimensional space and suppose that T has n distinct characteristic values. Prove that any linear operator which commutes with T is a polynomial in T.

8 For V the vectorspace of $n \times n$ matrices over a field F, let the linear operator T_A be defined for any fixed diagonal matrix $A \in V$ by $T_A(M) = AM - MA$. Show the family $\mathbf{F}$ of all linear operators T_A is simultaneously diagonalizable.

8 Let V be a finite-dimensional vectorspace and let W_1 be any subspace of V. Prove that there is a subspace W_2 of V such that $V = W_1 \oplus W_2$.

8 If $V = W_1 + \cdots + W_k$ and $\dim V = \dim W_1 + \cdots + \dim W_k$ then show the sum is direct.

8 Find a projection E such that $E(1, -1) = (1, -1)$ and $E(1, 2) = (0, 0)$.

8 If E_1 and E_2 are projections onto independent subspaces, then is $E_1 + E_2$ a projection?

8 If E is a projection and f is a polynomial, then $f(E) = aI + bE$. What are a and b in terms of the coefficients of f?

8 True or false: if a diagonalizable operator has only the characteristic values 0 and 1 it is a projection.

8 Prove that if E is the projection of V onto R along N then $(I - E)$ is the projection on N along R.

8 Let $E_1, \cdots, E_k$ be linear operators on V such that $I = E_1 + \cdots + E_k$.
(a) Prove that if $E_iE_j = 0$ when $i \neq j$, then $E_i = E_i^2$ for each i.
(b) Given that $E_1 + E_2 = I$ and the E_i are projections, show $E_1E_2 = 0 = E_2E_1$.

8 Let V be a real vectorspace and E an idempotent linear operator on V i.e. $E^2 = E$ i.e. a projection. Prove that $(E + I)$ is invertible and find its inverse.

8 Let F be a field of characteristic 0 and let V be an n-dimensional vectorspace over F. Let $E_1, \cdots, E_k$ be projections onto $W_1, \cdots W_k$ respectively having dimensions $\dim r_1, \cdots, \dim r_k$; and suppose that $E_1 + \cdots + E_k = I$. Prove $E_iE_j = 0$ when $i \neq j$.

8 For a linear operator $T : V \longrightarrow V$ on a finite dimensional vectorspace V, show $R \oplus N = V$ iff R and N are independent.

8 Let T be a linear operator on V. Suppose $V = W_1 \oplus \cdots \oplus W_k$ where each W_i is invariant under T. Let T_i be the restriction operator on W_i.
(a) Prove that $\det(T) = \det(T_1) \cdots \det(T_k)$
(b) Prove that the characteristic polynomial for f is the product of the characteristic polynomials for $f_1, \cdots, f_k$.
(c) Prove that the minimal polynomial for T is the LCM of the minimal polynomials for $T_1, \cdots, T_k$. Hint: prove and then use the corresponding facts about direct sums of matrices.

8 Let T be a linear operator on V that commutes with every projection on V. What can you say about T?