ME 597: UNCERTAINTY QUANTIFICATION

Instructors: Guang Lin, Ilias Bilionis

Lectures: 2 (75 min) lectures/week for 16 weeks

Textbook: None. The instructor will provide notes and weekly references.

Prerequisites: Working knowledge of probability and numerical methods for engineers at the level of Introduction to probability (MA 41600), and Numerical methods in mechanical engineering (ME 581).

UQ (8 Weeks)

1. Introduction (2 weeks)

- Probability theory. Random variables, expectations, conditional probabilities **(1 week)**
- Bayes rule, parameter estimation, model selection. **(1 week)**

2. Uncertainty Propagation and sensitivity analysis (4 weeks)

- Sampling methods. Monte Carlo, quasi-random sequences, Latin hypercube designs, multi-level Monte Carlo. **(1 week)**
- Classic collocation methods. Generalized polynomial chaos, sparse grid collocation. **(1 week)**
- Bayesian collocation methods. Bayesian linear regression, Gaussian process regression. **(1 week)**
- Sensitivity analysis **(1 week)**

3. Representation of prior uncertainty (2 weeks)

- Generic principles. Invariant probabilities, maximum entropy principle. **(0.5 week)**
- Dimensionality reduction. Principal component analysis, probabilistic principal component analysis. **(0.5 week)**
- Uncertainty in random fields. Gaussian random fields, covariance functions, Karhunen-Loève expansion of random fields. **(1 week)**

Calibration & data assimilation (5 weeks)

1. Calibration (4 weeks)

- Classical approach via minimization of (regularized) loss functions. Statistical (Bayesian) interpretation of the model calibration problem. **(1 week)**
- Sampling methods. Markov Chain Monte Carlo, Metropolis-Hastings, Sequential Monte Carlo. **(1 week)**
- Surrogate-based methods. Generalized polynomial chaos, Gaussian process regression, selection of experiments and simulations. (**1 week**)
- Variational methods. Relative entropy, Kullback-Leibler divergence. (**1 week**)

2. Data assimilation (1 week)

- Kalman filter, ensemble Kalman filter
- Generalized polynomial chaos based square-root Kalman filter
- Particle filter

Optimization (3 weeks)

Design optimization under uncertainty (3 weeks)

- Introduction. Robust optimization, risk quantification, multi-objective optimization, Pareto front, utility theory. **(0.5 week)**
- Sampling methods. Scenario based optimization, sampling average approximation. **(0.5 week)**
- Stochastic methods. Randomized search, Simulated Annealing method, Robins-Monro algorithm, particle swarm optimization. **(1 week)**
- Bayesian global optimization. Probability of improvement, expected improvement, knowledge gradient. **(1 week)**