

1. A solution of $\frac{dy}{dt} = \frac{2y}{t+1}$ with $y(1) = 8$ is
 - A. $y = (t+1)^2 + 4$
 - B. $y = 32(t+1)^{-2}$
 - C. $\boxed{y = 2(t+1)^2}$
 - D. $y = 4\sqrt{2(t+1)}$
 - E. $y = \sqrt{(t+1)^2 + 60}$

2. An implicit solution of $y' = \frac{2x}{y+x^2y}$ is
 - A. $\boxed{y^2 = 2\ln(1+x^2) + C}$
 - B. $y^2 = C\ln(1+x^2)$
 - C. $\frac{1}{2}y^2 = \ln x^2 + C$
 - D. $y^2 = \ln(1+x^2) + C$
 - E. $\frac{1}{2}y^2 = \ln|1+x| + C$

3. The substitution $v = y/x$ transforms the equation $\frac{dy}{dx} = \sin(y/x)$ into
 - A. $v' = \sin(v)$
 - B. $v' = x\sin(v)$
 - C. $v' + v = \sin(v)$
 - D. $\boxed{xv' + v = \sin(v)}$
 - E. $v' + xv = \sin(v)$

4. The solution in implicit form of

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$
 is:
 - A. $x^2 + y^2 = x^3 + C$
 - B. $\boxed{x^2 + y^2 = Cx^3}$
 - C. $x^2 + x^3 = y^2 + C$
 - D. $Cx^2 = x^3 + y^2$
 - E. $x^2 + y^3 + xy^2 = C$

5. Which of the following best describes the stability of equilibrium solutions for the autonomous differential equation $y' = y(4 - y^2)$?
 - A. $\boxed{y = 0 \text{ unstable; } y = 2 \text{ and } y = -2 \text{ both stable}}$

- B. $y = 0$ unstable; $y = 2$ stable
- C. $y = 0$ and $y = 2$ both stable
- D. $y = 0$ stable; $y = 2$ unstable; $y = -2$ stable
- E. $y = 0$ stable; $y = -2$ and $y = 2$ both unstable

6. Solve the initial value problem $y' - 2y = e^{-2t}$ with $y(0) = a$. For what value(s) of a is the solution bounded (i.e., not tending to infinity as $t \rightarrow +\infty$) on the interval $t > 0$?

$$\boxed{a = -1/4}$$

7. Solve the differential equation

$$(2xy + x^3)dx + (x^2 + y^3 + 2)dy = 0, \quad y(0) = 2.$$

$$\boxed{x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = 8.}$$

8. The function $y_1 = t^2$ is a solution of the differential equation

$$t^2 \frac{d^2y}{dt^2} - 2t \frac{dy}{dt} + 2y = 0.$$

Choose a function y_2 from the list below so that the pair y_1, y_2 form a fundamental set of solutions to the differential equation.

- A. $y_2 = \sin t$
- B. $y_2 = e^t$
- C. $y_2 = t \cos t$
- D. $\boxed{y_2 = t}$
- E. $y_2 = 2t^2$

9. A ball of mass 5 kg. is thrown upward with an initial velocity of 10 (m/sec). If we neglect the air resistance, the maximum height (in meters) that the ball can reach is: ($g = 9.8 \text{ m/sec}^2$ below)

- A. $\frac{100}{g}$
- B. $\boxed{\frac{50}{g}}$
- C. $50g$
- D. $\frac{10}{g}$
- E. $\frac{100}{5g}$

10. The largest open interval on which the solution to the initial value problem

$$(\cos t) y' + \frac{t}{t-3} y = \ln(4-t); \quad y(2) = 0$$

is guaranteed by the Existence and Uniqueness Theorem to exist is

A. $-\frac{\pi}{2} < t < \frac{\pi}{2}$

B. $0 < t < \pi$

C. $\frac{\pi}{2} < t < 3$

D. $2 < t < 4$

E. $4 < t < \infty$

11. The function $y_1(t) = t$ is a solution of the differential equation

$$t^2 \frac{d^2 y}{dt^2} + 2t \frac{dy}{dt} - 2y = 0, \quad t > 0.$$

Find another solution $y_2(t)$ such that y_1, y_2 form a set of fundamental solutions.

$$y_2(t) = t^{-2}$$

12. The solution of

$$y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

is:

$$y(t) = e^{-2t}(\cos t + 2 \sin t)$$