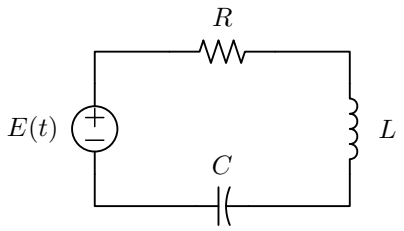
Computer Project 2. RLC Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: `ode45`, `plot`

Description: If $Q(t)$ = charge on a capacitor at time t in an RLC circuit (with R, L and C being the resistance, inductance and capacitance, respectively) and $E(t)$ = applied voltage, then Kirchhoff's Laws give the following 2nd order differential equation for $Q(t)$:

$$L\,Q''(t) + R\,Q'(t) + \frac{1}{C}\,Q(t) = E(t) \qquad (*)$$



Questions: Assume $L = 1$, $C = 1/5$, $R = 4$ and $E(t) = 10\cos\omega t$.

1. Use `ode45` (and plot routines) to plot the solution of $(*)$ with $Q(0) = 0$ and $Q'(0) = 0$ over the interval $0 \le t \le 80$ for $\omega = 0, 0.5, 1, 2, 4, 8, 16$.

2. Let $A(\omega)$ = maximum of $|Q(t)|$ over the interval $30 \le t \le 80$ (this approximates the amplitude of the steady-stat solution). Experiment with various values of ω and discuss what appears to happen to $A(\omega)$ as $\omega \to \infty$ and as $\omega \to 0$. Also, interpret your findings in terms of an equivalent spring-mass system.

Remark: There is an analogy between spring-mass system and RLC circuits given by:

Spring-mass system	**RLC circuit**
$m\,u'' + c\,u' + k\,u = F(t)$	$L\,Q'' + R\,Q' + \frac{1}{C}\,Q = E(t)$
$u =$ Displacement	$Q =$ Charge
$u' =$ Velocity	$Q' = I =$ Current
$m =$ Mass	$L =$ Inductance
$c =$ Damping constant	$R =$ Resistance
$k =$ Spring constant	$1/C =$ (Capacitance)$^{-1}$
$F(t) =$ External force	$E(t) =$ Voltage