1. A solution of $\dfrac{dy}{dt} = \dfrac{2y}{t+1}$ with $y(1) = 8$ is

A. $y = (t+1)^2 + 4$

B. $y = 32(t+1)^{-2}$

C. $\boxed{y = 2(t+1)^2}$

D. $y = 4\sqrt{2(t+1)}$

E. $y = \sqrt{(t+1)^2 + 60}$

2. An implicit solution of $y' = \dfrac{2x}{y + x^2 y}$ is

A. $\boxed{y^2 = 2\ln(1+x^2) + C}$

B. $y^2 = C\ln(1+x^2)$

C. $\dfrac{1}{2}y^2 = \ln x^2 + C$

D. $y^2 = \ln(1+x^2) + C$

E. $\dfrac{1}{2}y^2 = \ln|1+x| + C$

3. The substitution $v = \dfrac{y}{x}$ transforms the equation $\dfrac{dy}{dx} = \sin\left(\dfrac{y}{x}\right)$ into

A. $v' = \sin(v)$

B. $v' = x\sin(v)$

C. $v' + v = \sin(v)$

D. $\boxed{xv' + v = \sin(v)}$

E. $v' + xv = \sin(v)$

4. The solution in implicit form of

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

is:

A. $x^2 + y^2 = x^3 + C$

B. $\boxed{x^2 + y^2 = Cx^3}$

C. $x^2 + x^3 = y^2 + C$

D. $Cx^2 = x^3 + y^2$

E. $x^2 + y^3 + xy^2 = C$

5. Which of the following best describes the stability of equilibrium solutions for the autonomous differential equation $y' = y(4 - y^2)$?

A. $\boxed{y = 0 \text{ unstable; } y = 2 \text{ and } y = -2 \text{ both stable}}$

B. $y = 0$ unstable; $y = 2$ stable

C. $y = 0$ and $y = 2$ both stable

D. $y = 0$ stable; $y = 2$ unstable; $y = -2$ stable

E. $y = 0$ stable; $y = -2$ and $y = 2$ both unstable

6. Solve the initial value problem $y' - 2y = e^{-2t}$ with $y(0) = a$. For what value of a is the solution bounded (i.e., not tending to infinity as $t \to +\infty$)on the interval $t > 0$?

A. $a = 0$

B. $a = 1$

C. $a = -1$

D. $\boxed{a = -\frac{1}{4}}$

E. $a = \frac{1}{4}$

7. Solve the differential equation

$$(2xy + x^3)dx + (x^2 + y^3 + 2)dy = 0, \quad y(0) = 2.$$

A. $x^2y + 2y = 4$

B. $x^4 + 2y = 8$

C. $\boxed{x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = 8}$

D. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 = 0$

E. $\frac{1}{4}x^4 + \frac{1}{4}y^4 = 8$

8. The function $y_1 = t^2$ is a solution of the differential equation

$$t^2\frac{d^2y}{dt^2} - 2t\frac{dy}{dt} + 2y = 0.$$

A. $y_2 = \sin t$

B. $y_2 = e^t$

C. $y_2 = t\cos t$

D. $\boxed{y_2 = t}$

E. $y_2 = 2t^2$

9. A ball of mass 5 kg. is thrown upward with an initial velocity of 10 (m/sec). If we neglect the air resistance, the maximum height that the ball can reach is: ($g = 9.8$ m/sec^2)

A. $\frac{100}{g}$

B. $\boxed{\frac{50}{g}}$

C. $50g$

D. $\frac{10}{g}$

E. $\frac{20}{g}$

10. The largest open interval on which the solution to the initial value problem

$$\cos t y' + \frac{t}{t-3} y = \ln(4-t), \quad y(2) = 0$$

is guaranteed by the Existence and Uniqueness Theorem to exist is

A. $(-\frac{\pi}{2}, \frac{\pi}{2})$

B. $(0, \pi)$

C. $\boxed{(\frac{\pi}{2}, 3)}$

D. $(2, 4)$

E. $(4, \infty)$

11. The function $y_1 = t$ is a solution of the differential equation

$$t^2 \frac{d^2y}{dt^2} + 2t\frac{dy}{dt} - 2y = 0, \quad t > 0.$$

Find another solution $y_2(t)$ such that y_1, y_2 form a set of fundamental solutions.

A. $y_2 = t^2$

B. $\boxed{y_2 = t^{-2}}$

C. $y_2 = t^3$

D. $y_2 = t \ln t$

E. $y_2 = t^2 \ln t$

12. The solution of

$$y'' + 4y' - 5y = 0, \quad y(0) = 2, y'(0) = -4$$

is

A. $y = e^t$

B. $y = e^{-5t}$

C. $y = e^{-t} + e^{5t}$

D. $y = 2e^{-2t}$

E. $\boxed{y = e^t + e^{-5t}}$