

1. Find the solution of the initial value problem

$$y'' - y' + 0.25y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{3}.$$

A. $y = 2e^{t/2}$

B. $y = 2e^{t/2} - \frac{2}{3}te^{t/2}$

C. $y = \frac{2}{3}te^{t/2}$

D. $y = -2e^{t/2}$

E. $y = 2e^{t/2} + \frac{2}{3}te^{t/2}$

2. Find a particular solution of

$$y'' - 3y' - 4y = 2 \sin t.$$

A. $Y(t) = -\frac{5}{17} \sin t$

B. $Y(t) = \frac{3}{17} \cos t$

C. $Y(t) = -\frac{5}{17} \sin t + \frac{3}{17} \cos t$

D. $Y(t) = \frac{5}{17} \sin t$

E. $Y(t) = \frac{5}{17} \sin t + \frac{3}{17} \cos t$

3. Find a particular solution of

$$y'' - 3y' - 4y = 2e^{-t}.$$

A. $Y(t) = 2e^{-t}$

B. $Y(t) = 2t$

C. $Y(t) = te^{-t}$

D. $Y(t) = \frac{2}{5}e^{-t}$

E. $Y(t) = -\frac{2}{5}te^{-t}$

4. A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in. and then released, and if there is no damping, determine the position u of the mass at any time t .

A. $u(t) = \sin(2t)$

B. $u(t) = \cos(4t)$

C. $u(t) = \sin(2t) + \cos(4t)$

D. $u(t) = \frac{1}{4} \sin(\sqrt{2}t)$

E. $u(t) = \frac{1}{4} \cos(8t)$

5. Which of the following forms a fundamental set of solutions to the homogeneous differential equation $y^{(4)} - 2y'' + y = 0$.

- A. $\{\cos t, \sin t, e^t, e^{-t}\}$
- B. $\{e^t, te^t, e^{-t}, te^{-t}\}$
- C. $\{\cos t, t \sin t, t \cos t, \sin t\}$
- D. $\{e^t, e^{-t}\}$
- E. $\{e^t \cos t, e^t \sin t, e^{-t} \cos t, e^{-t} \sin t\}$

6. Given that $y_1(t) = t$ is a solution to

$$t^2 y'' + ty' - y = 0, \quad t > 0.$$

Find the general solution of the above equation.

- A. $c_1 t + c_2$
- B. $c_1 t + c_2 t^2$
- C. $c_1 t + \frac{1}{t}$
- D. $c_1 t + c_2 \frac{1}{t}$
- E. $c_1 t + t \ln t$

7. Which of the following set of functions is linearly independent?

- A. $f_1(t) = 1, \quad f_2(t) = t, \quad f_3(t) = t - 2$
- B. $f_1(t) = 1, \quad f_2(t) = t^2 + 1, \quad f_3(t) = t^2 - 1$
- C. $f_1(t) = 1, \quad f_2(t) = 2t - 3, \quad f_3(t) = t^2 + t + 1$
- D. $f_1(t) = 2t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t, \quad f_4(t) = t^2 + t + 1$
- E. $f_1(t) = 1, \quad f_2(t) = 2t - 3, \quad f_3(t) = 2t^2 + 1, \quad f_4(t) = 3t^2 + 1$

8. Find the general solution of

$$y^{(4)} - 5y'' + 4y = 0$$

- A. $y(t) = c_1 e^t + c_2 e^{-t}$
- B. $y(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-2t}$
- C. $y(t) = c_1 e^{2t} + c_2 e^{-2t}$
- D. $y(t) = c_1 e^t + c_2 e^{2t}$
- E. $y(t) = c_1 e^{-t} + c_2 e^{-2t}$

9. Find the general solution of

$$y^{(4)} + 2y'' + y = 0.$$

- A. $y(t) = c_1 \sin t + c_2 \cos t + c_3 t \sin t + c_4 t \cos t$
- B. $y(t) = c_1 \sin t + c_2 \cos t$
- C. $y(t) = c_1 \sin t + c_2 t \sin t$
- D. $y(t) = c_1 \cos t + c_2 t \cos t$
- E. $y(t) = c_1 \sin t + c_2 \cos t + c_3 t \cos t$

10. Find a particular solution of the equation

$$y''' - 4y' = t + 3 \cos t + e^{-2t}$$

- A. $y(t) = -t + \frac{1}{2} \cos t + e^{-2t}$
- B. $y(t) = t^2 + \frac{1}{2} \cos t - e^{-2t}$
- C. $y(t) = -t + \frac{1}{2} \sin t + \frac{1}{8} t e^{-2t}$
- D. $y(t) = -\frac{1}{8} t^2 - \frac{3}{5} \sin t + \frac{1}{8} t e^{-2t}$
- E. $y(t) = t^2 + \frac{1}{2} \cos t - t e^{-2t}$

11. Find the Laplace transform of

$$f(t) = 5e^{-2t} - 3 \sin 4t, \quad t \geq 0.$$

- A. $\mathcal{L}\{f(t)\} = \frac{5}{s+2} - 3 \sin 4t$
- B. $\mathcal{L}\{f(t)\} = \frac{12}{s^2+16} + 5e^{-2t}$
- C. $\mathcal{L}\{f(t)\} = \frac{5}{s^2+2} - \frac{12}{s^2+16}$
- D. $\mathcal{L}\{f(t)\} = \frac{5}{s+2} - \frac{12}{s^2+16}$
- E. $\mathcal{L}\{f(t)\} = \frac{5}{s+2} + \frac{12}{s^2+16}$

12. Find the Laplace transform $Y(s) = \mathcal{L}\{y\}$ of the solution of the initial value problem

$$y'' + y = 2 \sin t, \quad y(0) = 2, \quad y'(0) = 1.$$

- A. $Y(s) = \frac{2s}{s^2+1}$
- B. $Y(s) = \frac{5/3}{s^2+1}$

C. $Y(s) = \frac{2/3}{s^2 + 4}$

D. $Y(s) = \frac{2s}{s^2 + 1} + \frac{5/3}{s^2 + 1} - \frac{2/3}{s^2 + 4}$

E. $Y(s) = \frac{2s^3 + s^2 + 2s + 3}{(s^2 + 1)^2}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	e^{at}	$\frac{1}{s-a}$
3.	t^n	$\frac{n!}{s^{n+1}}$
4.	t^p ($p > -1$)	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\frac{a}{s^2 + a^2}$
6.	$\cos at$	$\frac{s}{s^2 + a^2}$
7.	$\sinh at$	$\frac{a}{s^2 - a^2}$
8.	$\cosh at$	$\frac{s}{s^2 - a^2}$
9.	$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
10.	$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
11.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), c > 0$
16.	$\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$