MATH 266 Quiz 3

Name: Solution PID:

Solve the problem systematically and show all your work.

(4pts)1. Given $y_1(t) = t$, use the method of reduction of order to find a second solution $y_2(t)$ of

$$t^2y'' + 2ty' - 2y = 0, \quad t > 0.$$

Sol:

$$\left.\begin{aligned} &\text{let } y(t) = v(t)y_1(t) = tv \\ &y' = v + tv' \\ &y'' = 2v' + tv'' \\ &t^2y'' + 2ty' - 2y = 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} t^3v'' + 4t^2v' = 0 \\ t > 0 \end{aligned}\right\} \Rightarrow tv'' + 4v' = 0$$

$$\text{let } u = v' \Rightarrow tu' = -4u \Rightarrow \frac{du}{u} = -4\frac{dt}{t} \Rightarrow u = kt^{-4}$$

$$\Rightarrow v' = kt^{-4} \Rightarrow v = \int kt^{-4}dt + c_1 = c_2t^{-3} + c_1$$

$$\left.\begin{aligned} &\Rightarrow y = tv = c_1t + c_2t^{-2} = c_1y_1(t) + c_2y_2(t) \\ &W(y_1, y_2)(t) = \begin{vmatrix} t & t^{-2} \\ 1 & -2t^{-3} \end{vmatrix} = -3t^{-2} \neq 0 \end{aligned}\right\} \Rightarrow \boxed{y_2(t) = t^{-2}}$$

(6pts)2. Find the general solution of

$$y'' + 2y' + 2y = 25te^t.$$

Sol: ① general solution to $y'' + 2y' + 2y = 0$

$$r^2 + 2r + 2 = 0 \Rightarrow r = -1 \pm i \Rightarrow y_c(t) = e^{-t}(c_1\cos t + c_2\sin t)$$

② $g(t) = 25te^t$

$$\left.\begin{aligned} &\text{let } Y(t) = (At+B)e^t \\ &Y'(t) = (At+A+B)e^t \\ &Y''(t) = (At+2A+B)e^t \\ &Y'' + 2Y' + 2Y = 25te^t \end{aligned}\right\}$$

$$\Rightarrow At + 2A + B + 2(At + A + B) + 2(At + B) = 25t$$

$$\left.\begin{aligned} &\Rightarrow 5A = 25 \Rightarrow A = 5 \\ &4A + 5B = 0 \end{aligned}\right\} \Rightarrow B = -4 \Rightarrow Y(t) = (5t-4)e^t$$

③ general solution to $y'' + 2y' + 2y = 25te^t$ is $\boxed{e^{-t}(c_1\cos t + c_2\sin t) + (5t-4)e^t}$