Chapter 3

Cavity Scattering

3.1 Introduction

The analysis of the electromagnetic scattering properties of cavities in a conducting ground plane is of high interest to the engineering community. Applications include the design of cavity-backed conformal antennas for civil and military use, and the characterization of radar cross section (RCS) of vehicles with grooves.

RCS is a measure of the detectability of an aircraft by radar systems. Methods of RCS reduction have been extensively studied over the past several decades to have found many practical applications including the design of modern stealth aircraft. Perhaps equally important are method of RCS enhancement, an example of which is the use of a string of corner reflectors to locate runways by approaching aircraft in bad weather. Mathematical and computational methods to accurately predict the RCS of complex objects such as aircraft are of great interest to designers. Of particular importance is the prediction of the RCS of cavities, both because cavity RCS can dominate the total aircraft RCS and because of its challenging computational nature. Examples of cavities include jet engine inlet ducts, cavity-backed antennas, and cracks and gaps in the metallic skin of the aircraft.

We consider a time-harmonic electromagnetic plane wave incident on an arbitrarily shaped open cavity embedded in an infinite ground plane. The ground plane and the walls of the open cavity are perfect electric conductors (PEC), and the interior of the open cavity is filled with a nonmagnetic material which may be inhomogeneous. The space above the ground is filled with a homogeneous, linear isotropic medium characterized by its permittivity ε_0 and permeability μ_0. Two fundamental polarizations, transverse electric (TE) and transverse magnetic (TM), are considered to study the propagation of the scattered waves from the cavity, and hence its RCS.

There is a large engineering literature available on computation and design of cavities. This chapter is devoted to a variational approach for solving the scattering problem. The fundamental step of the approach is to reduce the infinite nature of the scattering problem into a bounded domain (the cavity) problem by introducing a transparent boundary condition. In the general two-dimensional setting, the well-posedness will be established. The results indicate that the scattering problem in both TE and TM polarizations attains a unique weak solution for a general cavity medium. Computationally, the variational approach leads naturally to a class of finite element methods.

It is a common assumption that the cavity opening coincides with the aperture on an infinite ground plane, and hence simplifying the modeling of the exterior (to the cavity) domain. This limits the application of these methods since many cavity openings are not planar. We introduce a solid mathematical technique which is capable of characterizing the scattering by overfilled cavities in the frequency domain. In particular, we seek to determine the fields scattered by the protruding cavity upon a given incident wave. The method decomposes the entire solution domain to two sub-domains via an artificial semicircle enclosing the cavity: the infinite upper half plane over the perfect electrically conducting ground plane exterior to the semicircle, and the cavity plus the interior region. The problem is solved exactly in the infinite sub-domain, while the other is solved using finite elements. The two regions are coupled over the semicircle via the introduction of a boundary operator exploiting the field continuity over material interfaces.

3.2 Problem formulation

Throughout, the media are assumed to be non-magnetic, and a constant magnetic permeability, $\mu = \mu_0$, is assumed everywhere. Then the electromagnetic wave propagation is governed by the time-harmonic Maxwell equations (time dependence $e^{-i\omega t}$):

$$\operatorname{curl} \mathbf{E} = i\omega \mathbf{B}, \tag{3.1}$$

$$\operatorname{curl} \mathbf{H} = -i\omega \mathbf{D} + \mathbf{J}, \tag{3.2}$$

where $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field, $\mathbf{B}$ is the magnetic flux density, $\mathbf{D}$ is the electric flux density, $\mathbf{J}$ is the electric current density, and ω is the angular frequency. The constitutive relations, describing the macroscopic properties of the medium, are taken as

$$\mathbf{B} = \mu \mathbf{H}, \quad \mathbf{D} = \varepsilon \mathbf{E}, \quad \text{and} \quad \mathbf{J} = \sigma \mathbf{E},$$