Math 181 Recitation 10-25

Due at recitation, Thurs, Oct. 25, 2007

1. (a) Use l'Hôpital's rule (or draw a picture) to find $\lim_{x\to 0} \frac{\sin(x)}{x}$. Deduce that $\int_0^\pi \frac{\sin(x)}{x}\,dx$ converges.

(b) Show that for any integer $n \geq 0$,

$$\int_{n\pi}^{(n+1)\pi} \frac{|\sin(x)|}{x}\,dx > \int_{(n+\frac{1}{6})\pi}^{(n+\frac{5}{6})\pi} \frac{|\sin(x)|}{x}\,dx > \frac{1}{3}\frac{1}{n+1}.$$

Deduce that $\int_0^\infty \frac{|\sin(x)|}{x}\,dx$ diverges.

(c) Show that

$$\int_a^b \frac{\sin(x)}{x}\,dx = \frac{1-\cos(b)}{b} - \frac{1-\cos(a)}{a} + \int_a^b \frac{1-\cos(x)}{x^2}\,dx.$$

Deduce that $\int_0^\infty \frac{\sin(x)}{x}\,dx$ converges.

(d) The *Fresnel integral* $\int_0^\infty \sin(x^2)\,dx$ occurs in the theory of diffraction of light. Use the substitution $x^2 = u$ to transform this integral into one whose convergence can be shown as in (c). (And do the convergence argument for this transformed integral.)

(e) Show that the function $2x\cos(x^4)$ is unbounded on $[0,\infty)$, but that $\int_0^\infty 2x\cos(x^4)\,dx$ converges. (Use the same substitution as in (d)).

2. `p. 743, #90(a).` [Part (b) is optional.]

Note that $a_n \approx b_n$ just means that $\lim_{n\to\infty} a_n/b_n = 1$.