Math 181 Recitation 11-15

Due at recitation, Thurs, Nov. 15, 2007

1. Evaluate the sum

$$\sum_{n=0}^{\infty} \frac{1}{2n+1} \frac{1}{(-3)^n}.$$

2. Use the Taylor series for e^x to find the Taylor series (around $x = 0$) for $\sinh x$ and for $\cosh x$.

3. Find the Taylor series (around $x = 0$) for $\tanh^{-1} x$,

(a) by integrating the series for $1/(1 - x^2)$ term-by-term; and

(b) by using the Taylor series for $\ln(1 + x)$ and for $\ln(1 - x)$.

4. (a) Find integers b_0, b_1, b_2, b_3, such that

$$x^3 = b_0(x+1)(x+2)(x+3)/3! + b_1(x+1)(x+2)/2! + b_2(x+1)/1! + b_3.$$

(b) Use (a) and the Taylor series for $1/(1 - x)^r$ $(r = 1, 2, 3, 4)$ (binomial theorem) to evaluate $\sum_{n=0}^{\infty} n^3 x^n$ $(|x| < 1)$.

Remark. In a similar way one can find $\sum_{n=0}^{\infty} n^r x^n$ for any fixed r; and then one can find $\sum_{n=0}^{\infty} P(n)x^n$ for any polynomial P. For example,

$$\sum_{n=0}^{\infty} (an^2 + bn^3)x^n = a \sum_{n=0}^{\infty} n^2 x^n + b \sum_{n=0}^{\infty} n^3 x^n.$$