Math 182 Final Exam

April 29, 2008

NAME:____

There should be twelve pages.
Per question scoring: *correct,* 4; *incorrect,* −1; *blank,* 0. *No partials.*

1. A particle starts at the origin, with initial velocity $(1, 1, -1)$. Its acceleration is $(6t, 2, 6t)$. What is its position at time $t = 1$?

A. $(\frac{1}{6}, \frac{1}{2}, \frac{1}{3})$ B. $(\frac{7}{6}, \frac{1}{2}, \frac{-5}{6})$ C. $(1, 2, -1)$ D. $(3, 3, -5)$ E. $(2, 2, 0)$

2. Pick out a parametric representation of the tangent line at $(1, -2, 3)$ to the intersection of the surfaces $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$ and $2x + 3y + z = -1$.

A. $(1 + 9t, -2 - 2t, 3 + 24t)$ B. $(1 + 2t, -2 + 3t, 3 + t)$
C. $(1 + 27t, -2 + 6t, 3 + 72t)$ D. $(2 + t, 3 - 2t, 1 + 3t)$
E. $(28 + 9t, 4 + 2t, -69 - 24t)$

3. Suppose that the function $z = f(x, y)$ is such that $xe^y + ye^z = 0$. The equation of the tangent plane to the graph of $f(x, y)$ at the point $(-2, 2, 2)$ is:

A. $2X - 2Y + Z = -6$ B. $2X + 2Y - Z = -2$ C. $X + Y - 2Z = -4$
D. $X - Y + 2Z = 0$ E. None of the preceding.

4. Find all the local maxima, local minima, and saddle points of the function $f(x, y) = 4xy - x^4 - y^4$.

A. $(1, 1), (-1, 1)$ saddle points, $(0, 0)$ minimum.
B. $(1, 1), (-1, 1)$ maxima, $(0, 0)$ minimum.
C. $(1, 1)$ maximum, $(0, 0)$ saddle point, $(1, -1)$ minimum
D. $(1, 1), (-1, -1)$ maxima, $(0, 0)$ saddle point
E. None of the preceding.

5. Find the minimum distance from the origin of a point on the intersection of the surfaces $x^2 + 2y^2 + z^2 = 1$ and $x + y = 1$ (an ellipse).

A. 1 B. $5/\sqrt{3}$ C. 2 D. $2/3$ E. $\sqrt{5}/3$

6. Find the work done by the force $\mathbf{F} = (y \sin xy, x \sin xy)$ along the curve $x = \tan y/2$ $(0 \le y \le \pi)$ from the origin to $(1, \pi/2)$.

A. $\pi/6$ B. $\pi/2$ C. 0 D. 1 E. 2π

7. Use the substitution $u = x+y,\ v = x^2-y^2$ to evaluate $\iint_R (x+y)^2 dxdy$ where R is the region bounded by the curves $x + y = 2$, $x + y = 4$, $x = y$ and $x^2 - y^2 = 4$.

A. 1 B. $2\sqrt{2}$ C. 6 D. $4\sqrt{2}$ E. 12

8. Find the centroid of the bowl-shaped region bounded by the surfaces $z = 2$, $z = 3$ and $x^2 + y^2 = 9z^2 - 36$.

A. $(0, 0, \frac{225}{84})$ B. $(0, 0, \frac{8}{3})$ C. $(0, 0, \frac{56}{21})$ D. $(2.5, 2.5, \frac{56}{21})$ E. $(0, 0, \frac{6\pi}{7})$

9. Compute $\int_C (xy + e^{x^2})dx + (x^2 - \ln(1 + y))dy$ where C consists of the line segment from $(0, 0)$ to $(\pi, 0)$ plus the curve $y = \sin x,\ 0 \le x \le \pi$, oriented counterclockwise.

A. 1 B. $-\ln(2)$ C. e^2 D. $\pi/2$ E. π

10. Let S be the part of the paraboloid $z = 1/4 + x^2 + y^2$ lying between $z = 1/4$ and $z = 5/4$. Compute the surface integral $\iint_S z d\sigma$.

A. $(25\sqrt{5} - 1)\pi/40$ B. $4\pi/3$ C. $5\sqrt{5}\pi/8$ D. $8\sqrt{17}$
E. None of the preceding.

11. Find the outward flux of the vector field $(x^3, y^3, 12z)$ across the cylinder (including top and bottom) bounded by $x^2 + y^2 = 4$, $z = 0$, and $z = 1$.

A. 8π B. 24π C. 72π D. 108 E. 36π

12. Compute $\displaystyle\int_0^{1/16} \int_{y^{1/4}}^{1/2} \cos(16\pi x^5) dxdy.$

A. $1/(80\pi)$ B. $1/80$ C. $1/(16\pi)$ D. $\pi/80$ E. 80π