Math 182 Midterm Exam 1

Feb. 12, 2008

[**Bold numbers**] indicate points (**15** total).

For each question, show your work: no credit for guessing.

I. [**2**] Circle the equation of the plane passing through the points $(1, 2, 3)$ and $(3, 2, 1)$, and perpendicular to the plane $4x - y + 2z = 0$.

A. $X - Y + Z = 2$ B. $X + Z = 4$ C. $X + 2Y + Z = 8$
D. $X + 6Y + Z = 16$ E. $X + 14Y + Z = 32$

II. [**2**] Let P be the plane tangent to the surface $x^3 - y^6 = \sin(yz^2)$ at the point $(4, 2, 0)$. The (perpendicular) distance to P from the point $(1, 1, 4)$ is:

A. $\sqrt{17}$ B. 4 C. $\sqrt{20}$ D. 1/17
E. None of the preceding.

III. [**3**] A comet has an elliptical orbit with eccentricity e. Its speed at the orbital point nearest to the sun (perigee) is x times its speed at the orbital point farthest from the sun (apogee). Circle the correct value of x.

A. 1 B. e C. $\frac{1+e}{1-e}$ D. $\frac{1-e}{1+e}$ E. $\frac{(1+e)^2}{(1-e)^2}$

Hint. Under the usual assignment of polar coordinates with origin at the sun, the speed at these two points is given by $r d\theta/dt$, because $dr/dt = 0$. Assuming this, use Kepler's second law, as applied to the areas swept out by the line from the sun to the comet during a tiny time Δt when the comet is extremely close to one or the other of these two points, to deduce the ratio of the speeds in terms of the lengths of those two lines.

IV. Let x be a function of y and z such that

$$x^2y^3 + y^2z^3 + z^2x^3 = 1.$$

(a) **[2]** The gradient $\nabla_{(0,1)}\, x$ at $(y, z) = (0, 1)$ is:

A. $(3/2, 0)$ B. $(-2/3, 0)$ C. $(0, 2/3)$ D. $(0, -2/3)$

E. None of the preceding.

(b) **[2]** Which of the following is best as an estimate of the change in x when (y, z) moves 0.1 units starting at $(0, 1)$ and heading straight at $(3, -3)$?

A. -0.45 B. 0.4 C. 0.053 D. -0.04 E. -0.053

V. **[4]** Find the largest distance from the origin to a point lying both in the first quadrant (i.e., both coordinates nonnegative) and on the curve $x^3 + 2y^3 = 1$.

A. 1 B. $\sqrt{5}\,/\sqrt[3]{10}$ C. $\sqrt{6}$ D. $\sqrt[3]{2}$ E. $1/\sqrt[3]{2}$