Math 182 Recitation1-31

Due at recitation, Thurs. Jan. 31, 2008

1. p. 980, #66.

2. p. 980, #75.

3. p. 988, #32.

4. p. 988, #42.

5. (a) Let $f(x)$ be a one-to-one continuous function defined on a closed interval $[a, b]$. Assume $f(b) > f(a)$. Prove that f is *strictly increasing* (that is, $f(c) > f(d)$ for any $c > d$ in $[a, b]$).

Hint. Show that the continuous function $g(t) = f(tb + (1-t)c) - f(ta + (1-t)d)$ $(0 \leq t \leq 1)$ never takes the value 0; and then use the intermediate-value theorem to deduce that $f(c) - f(d)$ has the same sign as $f(b) - f(a)$.

(b) Prove that for *any* strictly increasing function f—continuous or not—the inverse function f^{-1} (see §7.1) is continuous.

Hint. To show continuity at a point $x = f(y)$ in the domain of f^{-1}, for any $\epsilon > 0$ take

$$\delta = \min\big(f(y+\epsilon) - f(y),\, f(y) - f(y-\epsilon)\big).$$