Math 182 Recitation2-14

Due at recitation, Thurs. Feb. 14, 2008

1. Google "two-body problem," and look through the first two items (Wikipedia and Eric Weisstein's page).

In the second item, r means $|\mathbf{r}|$, and $\hat{\mathbf{r}}$ means $\frac{\mathbf{r}}{r}$. Look at equations (13) and (14), and then use what you have learned about equation (15) to explain why the two bodies move in ellipses with one focus at the center of mass (defined by equation (1)).

Remarks. Earth has a gravitational effect on the sun, and so induces some motion of the sun around the common center of gravity. That center of gravity is so close to the center of the sun (since the sun is so massive compared to the earth), that the motion is quite small. However, if Earth were Jupiter-sized, and closer to the sun, like Mercury, then the resulting motion of the sun would be much more noticeable—it would seem to be wobbling around the common center of gravity, which would now be considerably farther from the center of the sun.

This is exactly how it has been determined in the past 10–15 years that some nearby (relatively speaking) stars have large planets orbiting not too far from them: astronomical instruments have become sensitive enough to notice the wobbling.

2. Write down a fourth order Taylor polynomial, centered at $(0, 0)$, for the function $\sin(x)\cos(y)$, in two ways:

(a) By multiplying together low-order terms in the Taylor series for $\sin(x)$ and $\cos(y)$.

(b) By using the Taylor formula for functions of two variables.

You should get the same answer via (a) or (b)!

3. Find the values of a and b for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

of least area containing the circle

$$(x-1)^2 + y^2 = 1.$$

<u>Hint</u>. Use the more-or-less obvious fact that this ellipse has to be tangent to the circle to get the constraint $a^2 - b^2a^2 + b^4 = 0$. Specifically, find a quadratic equation for the x-coordinate of an intersection point, and note that tangency means this equation has coincident roots.