Math 182 Recitation2-7
Due at recitation, Thurs. Feb. 7, 2008

1. p. 1031, #12.

2. p. 1032, #40.

3. p. 1032, #43.

4. (a) Let \( g(x, y, z) \) be a function whose gradient doesn’t vanish at any point on the surface \( g(x, y, z) = 0 \). Let \( Q = (a, b, c) \) be a point not on that surface. Let \( P \) be a point on the surface whose distance from \( Q \) is minimal, that is, \( \leq \) the distance \( P'Q \) for any other \( P' \) on the surface. Show that the line joining \( P \) and \( Q \) is perpendicular to the surface at \( P \). (In other words, the sphere with center \( Q \) and passing through \( P \) is tangent to the surface at \( P \).)

   One way to proceed is to see what the Lagrange multiplier method says about minimizing the function \( (x-a)^2 + (y-b)^2 + (z-c)^2 \) with \( x, y, z \) constrained to satisfy \( g(x, y, z) = 0 \).

   (b) Which point of the sphere \( x^2 + y^2 + z^2 = 1 \) has the greatest distance from \((1, 2, 3)\)?

   (c) In triangle \( ABC \) let the sides \( BC, AC, AB \) have lengths \( a, b, c \), respectively. For a point \( P \) in the interior, let \( x(P), y(P), \) and \( z(P) \) be the distances of \( P \) to \( BC, AC, \) and \( AB \), respectively. Show that for the point where \( x^2 + y^2 + z^2 \) is minimal, it holds that

   \[
   \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{2\Delta}{a^2 + b^2 + c^2}
   \]

   where \( \Delta \) is the area of the triangle.

   \textbf{Hint.} Begin by showing that for every \( P, ax + by + cz = 2\Delta \). Then minimize \( x^2 + y^2 + z^2 \) subject to this restraint. You can do this with Lagrange multipliers; but there’s an even easier way, using part (a).

5. Google “Lagrange multiplier” and also “Joseph Louis Lagrange.”