Math 503 Homework 2

Due Fri. Sept. 7

1. (a) Find the inverse of 22 in $\mathbb{Z}_{91}$.

(b) Solve the congruence $11x \equiv 12 \pmod{91}$. (It may help to use (a).)

2. Let $a, a_1, a_2, \ldots, a_n$ be integers.

(a) Prove by induction on n:

If $(a, a_1) = (a, a_2) = \cdots = (a, a_n) = 1$ then $(a, a_1 a_2 \ldots a_n) = 1$.

(b) With assumptions as in (a), prove that there is an integer x that is $\equiv 1 \pmod{a}$ and is $\equiv 0 \pmod{a_i}$ for all $i = 1, 2, \ldots, n$.

(c) (Chinese Remainder Theorem.) Suppose that $(a_i, a_j) = 1$ whenever $i \neq j$. For any integers x_i $(i = 1, 2, \ldots, n)$, prove that there is an integer x that is $\equiv x_i \pmod{a_i}$ for all i, and that any two such x differ by a multiple of $a_1 a_2 \ldots a_n$.

3. Prove, for any integers a, b, c, and any $n > 0$:

(a) $(a^n, b^n) = (a, b)^n$.

(b) If $(a, b) = 1$ and $ab = c^n$ then $a = u(a, c)^n$, where u is a unit.

Do the following problems in Clark:

4. $\mathbf{24}\boldsymbol{\alpha}$.

5. $\mathbf{24}\boldsymbol{\beta}$.

6. $\mathbf{23}\boldsymbol{\varepsilon}$. (<u>Hint</u>: use $\mathbf{24}\boldsymbol{\alpha}$.)