Math 553, Fall 2012

Instructor: Joseph Lipman, Math 750, 494-1994. < www.math.purdue.edu/~lipman >

Office hours: M 4:30–5:30, F 4–5. Other times by appointment only.

Grader: Manish Mishra, MA1035, mmishraATmath.purdue.edu Office hour: Tuesday, 2:00–3:00.

Grading: Weekly homework: 35%; midterm (early October): 25%; final: 40%.

Text: Dummit & Foote, *Abstract Algebra,* 3rd edition.

Other references (on reserve in Library):

M. Artin, *Algebra.*

A. Clark, *Elements of Abstract Algebra.*

T. Hungerford, *Algebra.*

N. Jacobson, *Basic Algebra I.*

S. Lang, *Algebra.*

There are numerous other Algebra texts in the library, with alternative treatments of material, illuminating examples, etc.

Historical development (on reserve in Library):

N. Bourbaki, *Elements of the History of Mathematics.*

Prerequisites:

Math 503, or familiarity with the material in Chapters 1–3 of D & F.

Basic concepts of linear algebra (vector space, basis, dimension . . .).

Course outline (approximate):

I. GROUPS $\approx$ 10 lectures.

Chapters 4 and 5.

II. RINGS $\approx$ 10 lectures.

Chapters 7–9.

III. FIELDS $\approx$ 20 lectures.

Chapters 13–14.8

Some things you will know when the course is over:

(1) A bit about how to analyze groups using subgroups. Application: finding all groups of a given order with a small number of prime factors.

(2) A glimpse of one the great achievements of 20th-century mathematics: the classification of all simple groups.

(3) Some arithmetic in new contexts (quadratic rings of integers, like the Gaussian integers). Application: given a positive integer n, in how many ways can you write a prime p as $p = x^2 + ny^2$.

(4) Why you can construct a regular polygon with 17 sides using only straightedge and compass, but not one with 7 sides.

(5) Something about polynomials over finite fields. Application: quadratic reciprocity.

(6) Galois theory: analyzing polynomials and their roots from a field and group theoretic viewpoint. Solution of polynomial equations by radicals.