Math 553 Homework 10

Due Fri. Nov. 9, 2012

1. Dummit and Foote, p. 545, Exercises 1–4. (Do all four.)

2. Dummit and Foote, p. 545, Exercise 6.

3. Prove that an integer $p > 1$ is prime if and only if $(p-1)! \equiv -1 \pmod{p}$.

Hint. See Dummit and Foote, p. 551, Exercise 6.

4. Read the *notes on separability* on the course webpage, and p. 547 in D&F. Then do problem 2 on p. 595 in D&F. (Justify your answer.)

5. (a) Prove that for any prime p and $n > 0$ the polynomial

$$\frac{(Y+1)^{p^n} - 1}{(Y+1)^{p^{n-1}} - 1}$$

is irreducible in $\mathbb{Q}[Y]$.

Hint. Look at this polynomial mod p to see that all its coefficients outside the highest degree term are divisible by p.

(b) Deduce that if a positive integer m is divisible by the square of an odd prime, then it is impossible to construct a regular m-gon with straightedge and compass.