OSCAR ZARISKI

Oscar Zariski was born in Kobrin, Belarus, in 1899. After attending, in 1918–1920, the University of Kiev, where he was largely self-educated (of necessity, in those unsettled times), he went to Rome and studied under the preeminent Italian geometers G. Castelnuovo, F. Enriques, and F. Severi. He received his doctorate at the University of Rome in 1924, and shortly thereafter came to the U.S.A. There ensued, first at Johns Hopkins and then at Harvard, sixty years of remarkable mathematical activity, lasting well into his eighties—an inspiring counterexample to the dictum that the domain of creative mathematics is reserved for the young.

Except for some early publications on set theory, which reflected the historical and philosophical interests of his teacher Enriques, he worked entirely in the field of algebraic geometry. His work was characterized by its concern with matters of fundamental importance; its profound effect on the field is universally recognized. His pioneering applications of the methods of abstract commutative algebra to questions in geometry set the stage for massive developments in both subjects.

Zariski's far-reaching influence was due in no small part to his qualities as a teacher. He projected the power which he exerted over mathematics, and which mathematics held over him—he was a vibrant embodiment of the ideal of a scholar-teacher, a model of devotion to the pursuit of the good mathematical life. Through the force of his personality, his interest in people, and his ability to communicate with and motivate others, he attracted and helped to develop the talents of many young mathematicians.

Among his students were a good proportion of the algebraic geometers educated in the United States during the 1950's and 60's. (See the volume dedicated to Zariski on the occasion of his seventieth birthday, Publ. Math. IHES. **36** (1969).) Of them, Abhyankar, Hironaka, and Lipman lectured at the Obergurgl working week on Resolution of Singularities which the present book commemorates, as did Teissier, who was closely associated with Zariski after 1970.

Zariski was honored with a number of high recognitions accorded by the American Mathematical Society: he was awarded the Cole Prize in Algebra in 1944, was invited to deliver the Colloquium Research Lectures at the summer meeting of the A.M.S. at Yale in 1947, served as President of the Society in 1969 and 1970, and was awarded the Steele prize for cumulative influence (1981). He received honorary doctorates from Holy Cross College (1959), Brandeis University (1965), Purdue University (1974), and Harvard University (1981). At the national level, he was awarded the Medal of Science by President Lyndon Johnson (1965), and the Wolf prize for lifetime achievement from the government of Israel (1982).

Zariski's Collected Papers were published in four volumes by the M.I.T. Press. In his preface to those volumes (see [Z3] for the last version) he gives his view of the major landmarks of his career. There is more to be found in David Mumford's appreciation of Zariski as a scientist and a person [M], and in Carol Parikh's full-length biography [P].

Here is a brief summary of Zariski's scientific work.

In his earlier papers, he dealt with topological questions on algebraic varieties, especially problems having to do with the fundamental group. At that time, he was much influenced by Solomon Lefschetz at Princeton. (Castelnuovo himself had advised Zariski that the methods of the Italian school, though they had had many successes, had reached their limit, and that the topological work of Lefschetz would be of great importance for the future of algebraic geometry.)

In his 1935 monograph *Algebraic Surfaces* he expounded masterfully the achievements of the Italian geometers. This book provided a most important link between the classical and modern theories; its value was such that a second updated edition was put out by Springer-Verlag in 1971 (reprinted 1995). The book was a turning point in Zariski's career—and hence in the development of algebraic geometry—as it brought him to the realization that the Italian theory, for all its exciting beauty, had no adequate mathematical foundation: few of the main results had been properly proved.

It had been recognized by some mathematicians, notably Krull and van der Waerden, that a suitable basis for geometry might be found in what was then called modern algebra, a subject which had been developed under the leadership of Emmy Noether and Emil Artin. Zariski began, ab initio, an intensive study of algebra. (He had the good fortune at that time of being able to attend a seminar held weekly at Princeton by Emmy Noether, who had recently emigrated from Germany.) His efforts bore fruit when he foresaw the geometric potentialities of Krull's valuation theory. He tested his ideas against the basic problem of resolution of singularities. This led him to the formulation of new and solid foundations for algebraic geometry, especially in its birational aspects, and to rigorous algebraic proofs for the resolution of singularities on three-dimensional varieties and for local uniformization on varieties of any dimension (over fields of characteristic zero). It was in these latter, previously unexplored, areas that his new methods revealed their scope, opening the way for the subsequent successes of Abhyankar and Hironaka.

Around the same time Zariski found important applications for Krull's theory of local rings, in the study of local properties of algebraic varieties. This work produced substantial additions to the theory, culminating in his 1951 memoir on holomorphic functions [Z2]. Emphasis on local rings became a basic feature of the theory of schemes; and that memoir is the birthplace of formal schemes.

While Zariski did a great deal of algebra, he always considered geometric intuition to be the prime source of his ideas; and indeed, powerful geometric insight is an essential feature of his work. So, although Krull was instrumental in the

development of crucial algebraic topics such as valuation theory and local algebra, he nevertheless made no direct contribution to algebraic geometry as such. Krull had anticipated the implications of algebra for geometry, but it remained for Zariski to realize them. Also van der Waerden had begun to write on geometry in abstract algebraic terms before Zariski, but he did not reach, as Zariski did, to the deeper questions which lie at the heart of the subject.

The interaction between geometry and commutative algebra benefited both fields. Some of the more immediate happy results of their merging were described in Zariski's 1950 address to the International Congress of Mathematicians [Z1]. He continued during the 1950's to contribute from the new point of view to classical geometrical theories—algebraic surfaces, linear systems on algebraic varieties, etc. Commutative algebra grew rapidly too; in the late 1950's, Zariski collaborated with Pierre Samuel on a two-volume treatise, *Commutative Algebra,* which remains today a useful reference for the subject.

From 1962 on, Zariski worked mainly (not exclusively) on the algebraic theory of equisingularity. His multiple papers on the subject take up most of the 650-page volume [Z3]. Several young researchers, especially in France, joined him in this enterprise. Unfortunately, with his passing from the scene, Zariski's particular vision of equisingularity has been rather neglected in the past fifteen years. The wider study of stratified spaces is active, and much progress has been made e.g., in the algebra and geometry associated with Whitney stratifications. But with his unique geometric point of view, Zariski bequeathed us a number of open and apparently deep problems which still promise to lead to significant enhancements of our understanding of algebraic and analytic varieties. In this, as in his other work, he extended the horizons of our knowledge.

References

[M] Mumford, D.: Oscar Zariski, 1899–1986, Notices Amer. Math. Soc. bf 33 (1986), 891–894. (See also the video "Oscar Zariski and his work," Amer. Math. Soc., Providence, 1988.)

[P] Parikh, C. A.: The Unreal Life of Oscar Zariski, Academic Press, San Diego, London, 1991. (See also review by R. Hartshorne, American Math. Monthly **99** (1992), 482–486.)

[Z1] Zariski, O.: The fundamental ideas of abstract algebraic geometry, Proc. Internat. Congress of Mathematicians, Cambridge, Mass., 1950.

[Z2] Zariski, O.: Theory and applications of holomorphic functions on algebraic vasrieties over arbitraty ground fields, Memoirs Amer. Math. Soc. **69** (1951), 1–90.

[Z3] Zariski, O.: Collected Papers, vol. IV, MIT Press, Cambridge, Mass., 1979.