MA 35100 (L. Lipshitz) FALL 2008 Exam 2 8:30 MWF

Name ______________________________

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(10 pts) 1. The reduced row echelon form of $A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 6 & 9 & 6 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & 4 & 9 & 1 & 2 \end{bmatrix}$ is $\begin{bmatrix} 1 & 2 & 0 & 5 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$T: \mathbb{R}^5 \to \mathbb{R}^4$ is given by $T(\vec{x}) = A\vec{x}$.

(a) Find a basis for the kernel of T.

(b) Find a basis for the image of T.

(a)A basis for Ker(T) is

(b)A basis for Im(T) is

(10 pts) 2. Consider the plane V in $\mathbb{R}^3$ determined by $2x_1 - 3x_2 + 4x_3 = 0$.

(a) $\mathfrak{B} = \left\{ \begin{bmatrix} 8 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} \right\}$ is a basis for V. If $[\vec{x}]_{\mathfrak{B}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, find $\vec{x}$.

$\vec{x} =$

(b) Find $a, b, c, d \in \mathbb{R}$ so that $\mathfrak{A} = \left\{ \begin{bmatrix} 3 \\ a \\ b \end{bmatrix}, \begin{bmatrix} 2 \\ c \\ d \end{bmatrix} \right\}$ is a basis for V and such that

$$\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}_{\mathfrak{A}} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$\mathfrak{A} = \left\{ \begin{bmatrix} 3 \\ \\ \end{bmatrix}, \begin{bmatrix} 2 \\ \\ \end{bmatrix} \right\}$

(10 pts) 3. Find a basis for the space of all 3×3 matrices $X = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ that satisfy

$$X \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} X.$$

What is the dimension of this space?

A basis is

dimension $=$

(10 pts) 4. Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 4x_2 \\ 4x_1 - 3x_2 \end{bmatrix}$$

Find the matrix A of T relative to the basis $\mathfrak{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$.

$A =$

(5 pts) 5. A, B and C are $n \times n$ matrices. A is similar to B and B is similar to C. Must A be similar to C?

Briefly explain your answer.

Yes	
No	

Explanation.

(5 pts) 6. T is a linear transformation from $\mathbb{R}^6$ to $\mathbb{R}^7$ with $\text{Ker}(T) = \{\vec{0}\}$. Must there be a linear transformation $U: \mathbb{R}^7 \to \mathbb{R}^6$ such that $U \circ T$ is the identity on $\mathbb{R}^6$ (i.e. $U(T(\vec{x})) = \vec{x}$ for all $\vec{x} \in \mathbb{R}^6$)?

Briefly explain your answer.

Yes	
No	

Explanation.