MATH 37500 **Review on the earlier parts of the course**

1. How many bitstrings of length 10 have exaclty four zeros?

2. What is the coefficient of $x^3y^6z^5$ in $(x+y+z)^{14}$? Explain in words why your answer is correct.

3. How many words of length 7 contain both "a" and "b"?

4. In how many ways can 6 men and 8 women be lined up such that men are not adjacent?

5. How many strings of 5 digits without repetitions contain 1 or 2 but not both?

6. In how many ways can one travel from $(0,0)$ to $(8,11)$ passing through $(4,7)$ while moving only East or North?

7. How many strings of length 13, composed of the letters m, n, p, q and no others, have exactly 3 p's and 4 q's?

8. How many words of length 6 are there when adjacent letters being the same is not allowed?

9. How many solutions are there in non-negative integers to $x+y+z+w=30$ if $5<x<10$ and $6\leq y$?

10. Find the probability of getting 3 of a kind but nothing better in a five card hand.

11. What is the probability that 3 people play poker against each other and each gets 4 of a kind?

12. On a die, 4 has probability $\frac{2}{7}$, all other numbers have probability $\frac{1}{7}$. On a second die, 3 has probability $\frac{2}{7}$ and all others have probability $\frac{1}{7}$. Find the chance of rolling a 7 with this pair of loaded dice.

13. Toss a coin 10 times. Assume that head shows with probability 0.55 in each toss. Find the probability of getting at least 2 heads in the 10 tosses.

14. Imagine a casino has the following game. Roll a fair die 3 times. You get \$ 27 if you roll at least two 2's. Otherwise you get nothing. Find the minimum price the casino should ask for playing this game.

15. Find the recurrence for bitstrings that contain 0.

16. Find a recurrence for making a row of colored tiles, colors being red, green, gray. What if red tiles cannot be adjacent? What are the initial conditions? (Note: how many strings are there of length zero?)

17. How many permutations of the English alphabet do contain "fish" but not "rat"?

18. Prove by induction that $3\cdot 11^n+2\cdot 6^n$ is divisible by 5.

19. Find a recurrence for the number of strings using the letters a, b, c, d, e that have neither "*cd*" nor "*dd*" in them. (Hint: start at the end.)

20. Solve $a_n = 4a_{n-1} - 4a_{n-2}$ with $a_0 = 3, a_1 = 4$.

21. Let f_n be the n-th Fibonacci number: $f_0 = 0, f_1 = 1, f_2 = 1, \ldots$ Prove that $f_1 + f_3 + f_5 + \ldots + f_{2n+1} = f_{2n+2}$.

22. Find the generating function for the sequence a_n where a_n is the sum of the squares $1^2 + \cdots + n^2$.

23. Find the generating function for the Fibonacci sequence.

24. Prove or disprove: $n^2 - 79n + 1601$ is a prime for all positive integers n.

25. Find all solutions to $n^2 - 3n + 3 = 0 \mod 7$. What about $n^2 - 3n + 5 = 0$?

26. Find an inverse of $6 \mod 53$.

27. How many numbers between 1 and 10000 are not divisible by any of 5, 7, 11?

28. Solve the simultaneous congruences

$$a \equiv 3 \mod 17$$
$$a \equiv 2 \mod 5$$
$$a \equiv 1 \mod 3.$$

29. Let p be a prime number, $p > 2$. Prove that $p(p-1)(p+1)$ is divisible by 24.

30. Find the number of integers between 1 and 300 (including the ends) whose gcd with 300 is 1. Repeat with "... is 2". (This is harder).

31. Find the gcd of 111 and 243 and express it as a linear combination of 111 and 243.

32. Solve the simultaneous recurrence $a_n = 3a_{n-1} + 2b_{n-1}, b_n = a_{n-1} + 2b_{n-1}, a_0 = 1, b_0 - 2$.

33. A triangular number is $t_n = 1 + 2 + ... + n$. Find a recurrence for the sequence a_n given by $a_n = t_1 + t_2 + ... + t_n$.

34. Solve the recurrence in the previous problem.