Review questions

1. How many bitstrings of length 10 have exactly four zeros?

2. What is the coefficient of $x^3y^6z^5$ in $(x+y+z)^{14}$? Explain in words why your answer is correct.

3. How many words of length 7 contain both 'a' and 'b'?

4. In how many ways can 6 men and 8 women be lined up such that men are not adjacent?

5. How many strings of 5 digits without repetitions contain 1 or 2 but not both?

6. In how many ways can one travel from (0,0) to (8,11) going only East or North, and while passing through (4,7) ?

7. How many strings of length 13, composed of the letters m, n, p, q and no others, have exactly 3 p's and 4 q's?

8. How many words of length 6 are there when adjacent letters being equal is not allowed?

9. How many solutions are there to $x+y+z+w = 30$ if $x$ is between 5 and 10 and $y$ is at least 6?

10. Find the probability of getting 3 of a kind but nothing better.

11. What is the probability that 2 people play poker against each other and both get 4 of a kind?

12. On a die, 4 has probability 2/7, all others have 1/7. On a second die, 3 has probability 2/7 and all others ahve 1/7. Find the chance of rolling a 7 with this pair of loaded dice.

13. Toss a coin 10 times. Assume that head shows with 55 % in each roll. Find the probability of getting at least 2 heads in the 10 rolls.

14. Imagine a casino has the following game.
Roll a fair die 3 times. You get $27 if you roll at least two 2's. Otherwise you get nothing.
Find the minimum price the casino should ask for playing this game.

15. Find the recurrence for bitstrings that contain 0.

16. Find a recurrence for making a row of colored tiles, colors being red, green, gray.
What if red tiles cannot be adjacent? What are the initial conditions? (Note: how many strings are there of length zero?)
17. How many permutations of the English alphabet do contain ``fish'' but not ``rat''?

18. Prove by induction that $3 \cdot 11^n + 2 \cdot 6^n$ is divisible by 5.

19. Find a recurrence for the number of strings using the letters $a,b,c,d$ that do not have ``cd'' nor ``dd'' in them. (Hint: start at the end.)

20. Solve $a_n = 4 \cdot a_{n-1} - 4 \cdot a_{n-2}$ with $a_0=3$, $a_1=4$.

21. Let $f_i$ be the $n$-th Fibonacci number: $f_0=0$, $f_1=1$, $f_2=1$, ... Prove that $f_1 + f_3 + f_5 + \ldots + f_{2n+1} = f_{2n+2}$.

22. Find the generating function for the sequence $a_n$ where $a_n$ is the sum of the squares $1^2+\ldots+n^2$.

23. Find the generating function for the Fibonacci sequence.

24. Page 472, number 27.


27. How many numbers between 1 and 10000 are not divisible by any of 5, 7, 11?