

4 A 3 (n)  $(A \wedge B \wedge C \wedge D) \vee (A \wedge B \wedge \neg C \wedge D) \vee (A \wedge B \wedge \neg C \wedge \neg D) \vee (A \wedge \neg B \wedge C \wedge D)$   
 $\vee (A \wedge \neg B \wedge C \wedge \neg D) \vee (A \wedge \neg B \wedge \neg C \wedge D) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D)$   
 $\vee (\neg A \wedge B \wedge C \wedge D) \vee (\neg A \wedge B \wedge \neg C \wedge D) \vee (\neg A \wedge B \wedge \neg C \wedge \neg D)$

5 D 8.  $(\forall x)(Ix \rightarrow (\exists y)(x+y=0)), (\forall x)(\forall y)(Ix \wedge Iy \wedge x \neq y \wedge (\exists z)(\exists w)(x+z=0 \wedge y+w=0) \rightarrow w \neq z),$   
 $(\forall x)(\forall y)(\forall z)(Ix \wedge Iy \wedge Iz \wedge y+x=0 \wedge z+(-x)=0 \rightarrow y+z=0)$

5 E 9.  $(\forall x)(\forall y)Rxy \rightarrow Q \otimes$

10.  $(\forall y)[(\exists x)Rxy \rightarrow Q \otimes y]$

11.  $(\forall x)[(\exists y)Pxy \rightarrow (\forall z)(Rz \rightarrow Qxyz)]$  (none)

- 5 F. 1. (iii)(iv); 2. (i); 3. (ii)(iv); 4. (v); 5. (i); 6. (ii); 7. (i); 8. (i); 9. (v); 10. (v); 11. (ii);  
 12. (iii)(iv); 13. (iii)(iv); 14. (v); 15. (v); 16. (i); 17. (v); 18. (iii)(iv) 19. (ii); 20. (ii)(iv);  
 21. (v); 22. (v); 23. (iii); 24. (v); 25. (iii)(iv); 26. (iii)(iv); 27. (v); 28. (v); 29. (v);  
 30. (iii).

8 A	11. {1}	1.	$(\forall x)(\forall y)(Dx \wedge Ey \rightarrow Fxy)$	<i>P</i>
	{2}	2.	$(\forall x)(\forall y)(Dx \wedge Fxy \rightarrow Gy)$	<i>P</i>
	{3}	3.	$(\exists x)Dx$	<i>P</i> (for <i>CP</i> )
	{4}	4.	<i>Dz</i>	<i>z P</i> (for <i>EP</i> )
	{1}	5.	$Dz \wedge Ex \rightarrow Fzx$	1 <i>US</i> [ <i>z</i>   <i>x</i> ][ <i>x</i>   <i>y</i> ]
	{1}	6.	$Dz \rightarrow (Ex \rightarrow Fzx)$	5 <i>T</i>
	{1, 4}	7.	$Ex \rightarrow Fzx$	<i>z</i> 4, 6 <i>T</i>
	{2}	8.	$Dz \wedge Fzx \rightarrow Gx$	2 <i>US</i> [ <i>z</i>   <i>x</i> ][ <i>x</i>   <i>y</i> ]
	{2}	9.	$Dz \rightarrow (Fzx \rightarrow Gx)$	8 <i>T</i>
	{2, 4}	10.	$Fzx \rightarrow Gx$	<i>z</i> 4, 9 <i>T</i>
	{1, 2, 4}	11.	$Ex \rightarrow Gx$	7, 10 <i>T</i>
	{1, 2, 3}	12.	$Ex \rightarrow Gx$	3, 4, 11 <i>EP</i> [ <i>z</i>   <i>x</i> ]
	{1, 2, 3}	13.	$(\forall x)(Ex \rightarrow Gx)$	12 <i>UG</i>
	{1, 2}	14.	$(\exists x)Dx \rightarrow (\forall x)(Ex \rightarrow Gx)$	3, 13 <i>CP</i>

8 A	12. {1}	1.	$(\forall x)(\forall y)(Jx \wedge \neg Ky \rightarrow Lxy)$	$P$
	{2}	2.	$(\exists x)[Jx \wedge (\forall y)(My \rightarrow \neg Lxy)]$	$P$
	{3}	3.	$Mx$	$x$ $P$ (for $CP$ )
	{4}	4.	$Jz \wedge (\forall y)(My \rightarrow \neg Lzy)$	$z$ $P$ (for $EP$ )
	{4}	5.	$(\forall y)(My \rightarrow \neg Lzy)$	$z$ 4 $T$
	{4}	6.	$Mx \rightarrow \neg Lzx$	$z$ 5 $US[x y]$
	{3,4}	7.	$\neg Lzx$	$x, z$ 3,6 $T$
	{1}	8.	$Jz \wedge \neg Kx \rightarrow Lzx$	1 $US[z x][x y]$
	{1,3,4}	9.	$\neg(Jz \wedge \neg Kx)$	$x, z$ 7,8 $T$
	{1,3,4}	10.	$\neg Jz \vee Kx$	$x, z$ 9 $T$
	{1,3,4}	11.	$Kx$	$x$ 4,10 $T$
	{1,4}	12.	$Mx \rightarrow Kx$	3,11 $CP$
	{1,2}	13.	$Mx \rightarrow Kx$	2,4,12 $EP[z x]$
	{1,2}	14.	$(\forall x)(Mx \rightarrow Kx)$	13 $UG$

8 A	13. {1}	1.	$(\exists x)[Qx \wedge (\forall y)(Py \rightarrow Rxy)]$	$P$
	{2}	2.	$(\forall x)[Px \rightarrow (\forall y)(Qy \wedge Sy \rightarrow \neg Ryx)]$	$P$
	{3}	3.	$(\exists x)Px$	$P$ (for $CP$ )
	{4}	4.	$Pu$	$u$ $P$ (for $EP$ )
	{5}	5.	$Qv \wedge (\forall y)(Py \rightarrow Rvy)$	$v$ $P$ (for $EP$ )
	{5}	6.	$(\forall y)(Py \rightarrow Rvy)$	$v$ 5 $T$
	{5}	7.	$Pu \rightarrow Rvu$	$v$ 6 $US[u y]$
	{4,5}	8.	$Rvu$	$u, v$ 4,7 $T$
	{2}	9.	$Pu \rightarrow (\forall y)(Qy \wedge Sy \rightarrow \neg Ryu)$	2 $US[u x]$
	{2,4}	10.	$(\forall y)(Qy \wedge Sy \rightarrow \neg Ryu)$	$u$ 4,9 $T$
	{2,4}	11.	$Qv \wedge Sv \rightarrow \neg Rvu$	$u$ 10 $US[v y]$
	{2,4}	12.	$\neg(Qv \wedge Sv)$	$u, v$ 8,11 $T$
	{2,4}	13.	$\neg Qv \vee \neg Sv$	$u, v$ 12 $T$
	{2,4,5}	14.	$\neg Sv$	$v$ 5,13 $T$
	{2,3,5}	15.	$\neg Sv$	$v$ 3,4,14 $EP[u x]$
	{2,3,5}	16.	$(\exists x)(\neg Sx)$	15 $EG[v x]$
	{1,2,3}	17.	$(\exists x)(\neg Sx)$	1,5,16 $EP[v x]$
	{1,2}	18.	$(\exists x)Px \rightarrow (\exists x)(\neg Sx)$	3,17 $CP$

8 D

7. {3}	3.	$(\exists x)(Px \wedge Qx)$	$P$ (for CP)
{4}	4.	$Pz \wedge Qz$	$z$ $P$ (for EP)
{1}	5.	$Pz \rightarrow (\forall y)(Qy \rightarrow \neg Rzy)$	1 US[z x]
{1,4}	6.	$(\forall y)(Qy \rightarrow \neg Rzy)$	$z$ 4,5 T
{1,4}	7.	$Qz \rightarrow \neg Rzz$	$z$ 6 US[z y]
{1,4}	8.	$\neg Rzz$	$z$ 4,7 T
{2}	9.	$Pz \rightarrow (\forall y)(Sy \rightarrow Rzy)$	2 US[z x]
{2,4}	10.	$(\forall y)(Sy \rightarrow Rzy)$	$z$ 4,9 T
{2,4}	11.	$Sz \rightarrow Rzz$	$z$ 10 US[z y]
{1,2,4}	12.	$\neg Sz$	$z$ 8,11 T
{1,2,4}	13.	$(\exists y)\neg Sy$	12 EG[z y]
{1,2,3}	14.	$(\exists y)\neg Sy$	3,4,13 EP[z x]
{1,2}	15.	$(\exists x)(Px \wedge Qx) \rightarrow (\exists y)\neg Sy$	3,14 CP

8 D

8. {5}	5.	$(\forall x)(Px \rightarrow Qx)$	$P$ (for CP)
{6}	6.	$Pz \wedge (\forall y)(Sy \rightarrow Rzy)$	$z$ $P$ (for EP)
{5}	7.	$Pz \rightarrow Qz$	5 US[z x]
{2}	8.	$Qz \rightarrow Sz$	2 US[z x]
{2,5,6}	9.	$Sz$	$z$ 6,7,8 T
{6}	10.	$(\forall y)(Sy \rightarrow Rzy)$	$z$ 6 T
{6}	11.	$Sz \rightarrow Rzz$	10 US[z y]
{2,5,6}	12.	$Rzz$	$z$ 9,11 T
{2,5,6}	13.	$Rzz \wedge Sz$	$z$ 9,12 T
{2,5,6}	14.	$(\exists x)(Rxx \wedge Sx)$	13 EG[z x]
{2,6}	15.	$(\forall x)(Px \rightarrow Qx) \rightarrow (\exists x)(Rxx \wedge Sx)$	5,14 CP
{1,2}	16.	$(\forall x)(Px \rightarrow Qx) \rightarrow (\exists x)(Rxx \wedge Sx)$	1,6,15 EP[z x]
{17}	17.	$(\exists x)(Rxx \wedge Sx)$	$P$ (for CP)
{18}	18.	$Ruu \wedge Su$	$u$ $P$ (for EP)
{3}	19.	$Ruu \wedge Su \rightarrow Qu$	3 US[u x]
{3,18}	20.	$Qu$	$u$ 18,19 T
{4}	21.	$Qu \rightarrow (\forall y)(Qy \wedge Ruy)$	4 US[u x]
{3,4,18}	22.	$(\forall y)(Qy \wedge Ruy)$	$u$ 20,21 T
{3,4,18}	23.	$Qx \wedge Rux$	$u$ 22 US[x y]
{3,4,18}	24.	$Qx$	23 T
{3,4,18}	25.	$Px \rightarrow Qx$	24 T
{3,4,18}	26.	$(\forall x)(Px \rightarrow Qx)$	25 UG
{3,4,17}	27.	$(\forall x)(Px \rightarrow Qx)$	17,18,26 EP[u x]
{3,4}	28.	$(\exists x)(Rxx \wedge Sx) \rightarrow (\forall x)(Px \rightarrow Qx)$	17,27 CP
{1,2,3,4}	29.	$(\forall x)(Px \rightarrow Qx) \leftrightarrow (\exists x)(Rxx \wedge Sx)$	16,28 T

9 A

15.  $(\forall x)(\forall y)(\forall z)(Gx \wedge Py \wedge Ez \rightarrow Sxyz), (\forall x)[(\exists y)(\exists z)(Py \wedge Ez \wedge Sxyz) \rightarrow Dx],$   
 $(\forall y)(\forall z)(Py \wedge Ez \rightarrow Smyz) \therefore Gm \rightarrow Dm$

Not Valid.  $T = \{U, G, P, E, D, S, m\}, U = \{m\}.$

	G	D	P
m	T	F	F

9 A 16.  $(\forall x)[(\exists y)(\exists z)(Cy \wedge Mz \wedge Axyz) \rightarrow Wx \wedge Px] \therefore (\exists x)(\exists y)(\exists z)(Wx \wedge Cy \wedge Mz \wedge Axyz) \rightarrow$   
 $(\exists x)(Wx \wedge Px)$

Valid.

{1}	1.	$(\forall x)[(\exists y)(\exists z)(Cy \wedge Mz \wedge Axyz) \rightarrow Wx \wedge Px]$	P
{2}	2.	$(\exists x)(\exists y)(\exists z)(Wx \wedge Cy \wedge Mz \wedge Axyz)$	P(for CP)
{3}	3.	$(\exists y)(\exists z)(Wu \wedge Cy \wedge Mz \wedge Auyz)$	u P(for EP)
{4}	4.	$(\exists z)(Wu \wedge Cv \wedge Mz \wedge Auvs)$	u, v P(for EP(v))
{5}	5.	$Wu \wedge Cv \wedge Mw \wedge Auvs$	u, v, w P(for EP(w))
{5}	6.	$Cv \wedge Mw \wedge Auvs$	u, v, w 5 T
{5}	7.	$(\exists y)(\exists z)(Cy \wedge Mz \wedge Auyz)$	u 6 EG[v y][w z]
{1}	8.	$(\exists y)(\exists z)(Cy \wedge Mz \wedge Auyz) \rightarrow Wu \wedge Pu$	1 US[u x]
{1,5}	9.	$Wu \wedge Pu$	u 7,8 T
{1,5}	10.	$(\exists x)(Wx \wedge Px)$	9 EG[u x]
{1,4}	11.	$(\exists x)(Wx \wedge Px)$	4,5,10 EP[w z]
{1,3}	12.	$(\exists x)(Wx \wedge Px)$	3,4,11 EP[v y]
{1,2}	13.	$(\exists x)(Wx \wedge Px)$	2,3,12 EP[u x]
{1}	14.	$(\exists x)(\exists y)(\exists z)(Wx \wedge Cy \wedge Mz \wedge Axyz) \rightarrow$ $(\exists x)(Wx \wedge Px)$	2,13 CP

9 B

15. Valid.

{1}	1.	$(\exists x)(Jx \wedge Kx)$	P
{2}	2.	$(\exists z)[Jz \wedge (\forall y)(Ky \rightarrow \neg Lzy)]$	P
{3}	3.	$Ju \wedge Ku$	u P(for EP)
{4}	4.	$Jv \wedge (\forall y)(Ky \rightarrow \neg Lvy)$	v P(for EP)
{4}	5.	$(\forall y)(Ky \rightarrow \neg Lvy)$	v 4 T
{4}	6.	$Ku \rightarrow \neg Lvu$	v 5 US[u y]
{3,4}	7.	$\neg Lvu$	u, v 3,6 T
{3,4}	8.	$Jv \wedge \neg Lvu$	u, v 4,7 T
{3,4}	9.	$(\exists z)(Jz \wedge \neg Lzu)$	u 8 EG[v z]
{2,3}	10.	$(\exists z)(Jz \wedge \neg Lzu)$	u 2,4,9 EP[v z]
{2,3}	11.	$Ju \wedge (\exists z)(Jz \wedge \neg Lzu)$	u 3,10 T
{2,3}	12.	$(\exists x)[Jx \wedge (\exists z)(Jz \wedge \neg Lzx)]$	11 EG[u x]
{1,2}	13.	$(\exists x)[Jx \wedge (\exists z)(Jz \wedge \neg Lzx)]$	1,3,12 EP[u x]

9 C 10.  $Lxy$ :  $x$  likes  $y$ ,  $a$ : Arthur,  $b$ : Betty.

$(\forall x)(Lxb \rightarrow \neg Lxa)$ ,  $(\forall x)(\exists y)Lxy$ ,  $(\forall x)Lxa$

$S = \langle U, L, a, b \rangle$ ,  $U = \{a, b\}$ .

L	a	b
a	T	F
b	T	F

9 D 18. Inconsistent.

{1}	1.	$(\forall x)(\forall y)(Px \wedge Py \rightarrow Pf(x, y))$	$P$
{2}	2.	$(\forall x)(Px \rightarrow \neg Qx)$	$P$
{3}	3.	$(\exists x)[Px \wedge (\forall y)(Py \rightarrow Qf(y, x))]$	$P$
{4}	4.	$Pv \wedge (\forall y)(Py \rightarrow Qf(y, v))$	$v P(\text{for } EP)$
{4}	5.	$(\forall y)(Py \rightarrow Qf(y, v))$	$v 4 T$
{4}	6.	$Pv \rightarrow Qf(v, v)$	$v 5 US[v y]$
{4}	7.	$Qf(v, v)$	$v 4, 6 T$
{2}	8.	$Pf(v, v) \rightarrow \neg Qf(v, v)$	$2 US[f(v, v) x]$
{2, 4}	9.	$\neg Pf(v, v)$	$v 7, 8 T$
{1}	10.	$Pv \wedge Pv \rightarrow Pf(v, v)$	$1 US[v x][v y]$
{1, 4}	11.	$Pf(v, v)$	$v 4, 10 T$
{1, 2, 4}	12.	$Pf(v, v) \wedge \neg Pf(v, v)$	$v 9, 11 T$
{1, 2, 4}	13.	$A \wedge \neg A$	$12 T$
{1, 2, 3}	14.	$A \wedge \neg A$	$3, 4, 13 EP[v x]$

9 D 19. Consistent.  $S = \langle U, P, R, f \rangle$ ,  $U = Z$ ,  $f(x) = -x$ ,  $Px : x > 0$ ,  $Rxy : x \cdot y > 0$ .