

Chapter 6

The parts of the interpretations not specified can be defined arbitrarily.

- A.** 10. $(\forall x)(\forall y)[Px \wedge Qy \rightarrow (\exists z)(Rxy \wedge Sxyz)]; \quad S = \langle U, P, Q, R, S \rangle$

$$\text{true: } U = \{a\} \quad \frac{\quad}{a} \begin{array}{|c|c|} \hline P & Q \\ \hline F & F \\ \hline \end{array}$$

$$\text{false: } U = \{a\} \quad \frac{\quad}{a} \begin{array}{|c|c|} \hline P & Q \\ \hline T & T \\ \hline \end{array} \quad \frac{R}{a} \begin{array}{|c|} \hline a \\ \hline F \\ \hline \end{array}$$

11. $(\exists x)(\forall y)[Px \wedge Ry \wedge Sxy \wedge (\exists z)(Px \wedge Rx \wedge \neg Qxaz)]; \quad S = \langle U, P, Q, R, S, a \rangle$

$$\text{true: } U = \{a\} \quad \frac{\quad}{a} \begin{array}{|c|c|} \hline P & R \\ \hline T & T \\ \hline \end{array} \quad \frac{S}{a} \begin{array}{|c|} \hline a \\ \hline T \\ \hline \end{array} \quad Qaaa \text{ false}$$

$$\text{false: } U = \{a\} \quad \frac{\quad}{a} \begin{array}{|c|} \hline P \\ \hline F \\ \hline \end{array}$$

- B.** 10. $(\exists x)[Ex \wedge (\forall y)(Fy \rightarrow Gxy)], (\forall x)(\forall y)[Ex \rightarrow (Gxy \leftrightarrow Hy)];$
 $(\forall x)(Hx \leftrightarrow Fx); S = \langle U, E, F, G, H \rangle$

$$U = \{a\} \quad \frac{\quad}{a} \begin{array}{|c|c|c|} \hline E & F & H \\ \hline T & F & T \\ \hline \end{array} \quad \frac{G}{a} \begin{array}{|c|} \hline a \\ \hline T \\ \hline \end{array}$$

16. $(\forall x)[Ax \wedge Bx \rightarrow (\forall y)(By \wedge Cy \leftrightarrow Dxy)]; (\exists x)[Ax \wedge Bx \wedge (\forall y)(By \wedge Ey \leftrightarrow \neg Dxy)];$
 $(\exists x)(Ax \wedge Bx \wedge \neg Ex); S = \langle U, A, B, C, D, E \rangle$

$$U = \{a, b\} \quad \frac{\quad}{\begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array}} \begin{array}{|c|c|c|c|} \hline A & B & C & E \\ \hline T & T & F & T \\ \hline F & T & F & T \\ \hline \end{array} \quad \frac{D}{\begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array}} \begin{array}{|c|c|} \hline a & b \\ \hline F & F \\ \hline F & F \\ \hline \end{array}$$

Chapter 7

A. 11.	{1}	1.	$(\forall x)(\forall y)(Ax \wedge By \rightarrow \neg Rxy)$	P
	{2}	2.	Aa	P
	{3}	3.	$(\forall y)(Cy \rightarrow Ray)$	P
	{4}	4.	Cz	$z P(\text{for } CP)$
	{1}	5.	$Aa \wedge Bz \rightarrow \neg Raz$	1 $US[a x][z y]$
	{3}	6.	$Cz \rightarrow Raz$	3 $US[z y]$
	{3, 4}	7.	Raz	$z 4, 6T$
	{1, 3, 4}	8.	$\neg(Aa \wedge Bz)$	$z 5, 7 T$
	{1, 3, 4}	9.	$\neg Aa \vee \neg Bz$	$z 8 T$
	{1, 2, 3, 4}	10.	$\neg Bz$	$z 2, 9 T$
	{1, 2, 3}	11.	$Cz \rightarrow \neg Bz$	4, 10 CP
	{1, 2, 3}	12.	$(\forall z)(Cz \rightarrow \neg Bz)$	11 UG

12.	{1}	1.	$(\forall x)(Ax \rightarrow \neg Rxa)$	P
	{2}	2.	$Ab \wedge Ba$	P
	{3}	3.	$(\forall x)(Cx \rightarrow Bx)$	P
	{4}	4.	$(\forall x)(Ax \rightarrow (\forall y)(By \wedge Cy \rightarrow Rxy))$	P
	{1}	5.	$Ab \rightarrow \neg Rba$	1 $US[b x]$
	{1, 2}	6.	$\neg Rba$	2, 5 T
	{4}	7.	$Ab \rightarrow (\forall y)(By \wedge Cy \rightarrow Rby)$	4 $US[b x]$
	{2, 4}	8.	$(\forall y)(By \wedge Cy \rightarrow Rby)$	2, 7 T
	{2, 4}	9.	$Ba \wedge Ca \rightarrow Rba$	8 $US[a y]$
	{1, 2, 4}	10.	$\neg(Ba \wedge Ca)$	6, 9 T
	{1, 2, 4}	11.	$\neg Ba \vee \neg Ca$	10 T
	{1, 2, 4}	12.	$\neg Ca$	2, 11 T

B. 10. $(\forall x)(Mx \wedge Ax \rightarrow Bx \wedge Fx)$, $(\forall x)(Ax \wedge Bx \rightarrow Mx \wedge Fx)$, $(\forall x)(\neg Ax \rightarrow Mx \wedge Bx)$
 $\therefore (\forall x)(Mx \leftrightarrow Bx)$

{1}	1.	$(\forall x)(Mx \wedge Ax \rightarrow Bx \wedge Fx)$	P
{2}	2.	$(\forall x)(Ax \wedge Bx \rightarrow Mx \wedge Fx)$	P
{3}	3.	$(\forall x)(\neg Ax \rightarrow Mx \wedge Bx)$	P
{4}	4.	Mx	$x P(\text{for } CP)$
{5}	5.	$\neg Bx$	$x P(\text{for } IP)$
{5}	6.	$\neg Mx \vee \neg Bx$	$x 5 T$
{5}	7.	$\neg(Mx \wedge Bx)$	$x 6 T$
{3}	8.	$\neg Ax \rightarrow Mx \wedge Bx$	$3 US[x x]$
{3, 5}	9.	Ax	$x 7, 8 T$
{1}	10.	$Mx \wedge Ax \rightarrow Bx \wedge Fx$	$1 US[x x]$
{1, 3, 4, 5}	11.	$Bx \wedge Fx$	$x 4, 9, 10 T$
{1, 3, 4, 5}	12.	$Bx \wedge \neg Bx$	$x 5, 11 T$
{1, 3, 4}	13.	Bx	$x 5, 12 IP$
{1, 3}	14.	$Mx \rightarrow Bx$	$4, 13 CP$
{15}	15.	Bx	$x P(\text{for } CP)$
{16}	16.	Ax	$x P(\text{for } CP)$
{2}	17.	$Ax \wedge Bx \rightarrow Mx \wedge Fx$	$2 US[x x]$
{2, 15, 16}	18.	$Mx \wedge Fx$	$x 15, 16, 17 T$
{2, 15, 16}	19.	Mx	$x 18 T$
{2, 15}	20.	$Ax \rightarrow Mx$	$x 16, 19 CP$
	21.	$Mx \wedge Bx \rightarrow Mx$	T
{3}	22.	$\neg Ax \rightarrow Mx$	$8, 21 T$
{2, 3, 15}	23.	Mx	$x 20, 22 T$
{2, 3}	24.	$Bx \rightarrow Mx$	$15, 23 CP$
{1, 2, 3}	25.	$Mx \leftrightarrow Bx$	$14, 24 T$
{1, 2, 3}	26.	$(\forall x)(Mx \leftrightarrow Bx)$	$25 UG$

(On lines 16-23, we used an alternative proof.)

11. $(\forall x)(Fx \vee Mx \rightarrow (\forall y)(Cy \rightarrow Lxy)), Fn \wedge \neg Lnp, (\forall x)(Mx \rightarrow \neg Lxs),$
 $Mr \therefore \neg Cp \wedge \neg Cs$

{1}	1.	$(\forall x)(Fx \vee Mx \rightarrow (\forall y)(Cy \rightarrow Lxy))$	P
{2}	2.	$Fn \wedge \neg Lnp$	P
{3}	3.	$(\forall x)(Mx \rightarrow \neg Lxs)$	P
{4}	4.	Mr	P
{3}	5.	$Mr \rightarrow \neg Lrs$	3 $US[r x]$
{3, 4}	6.	$\neg Lrs$	4, 5 T
{1}	7.	$Fr \vee Mr \rightarrow (\forall y)(Cy \rightarrow Lry)$	1 $US[r x]$
{4}	8.	$Fr \vee Mr$	4 T
{1, 4}	9.	$(\forall y)(Cy \rightarrow Lry)$	7, 8 T
{1, 4}	10.	$Cs \rightarrow Lrs$	9 $US[s y]$
{1, 3, 4}	11.	$\neg Cs$	6, 10 T
{1}	12.	$Fn \vee Mn \rightarrow (\forall y)(Cy \rightarrow Lny)$	1 $US[n x]$
{2}	13.	$Fn \vee Mn$	2 T
{1, 2}	14.	$(\forall y)(Cy \rightarrow Lny)$	12, 13 T
{1, 2}	15.	$Cp \rightarrow Lnp$	14 $US[p y]$
{1, 2}	16.	$\neg Cp$	2, 15 T
{1, 2, 3, 4}	17.	$\neg Cp \wedge \neg Cs$	11, 16 T

D. 10.

{1}	1.	$(\forall x)(\forall y)(\forall z)(Rxy \wedge Ryz \rightarrow Rxz)$	P
{2}	2.	$(\forall x)(\forall y)(Rxy \rightarrow Ryx)$	P
{3}	3.	Rab	P
{4}	4.	$\neg Rbb$	P
{2}	5.	$Rab \rightarrow Rba$	2 $US[a x][b y]$
{2, 3}	6.	Rba	3, 5 T
{1}	7.	$Rba \wedge Rab \rightarrow Rbb$	1 $US[b x][a y][b z]$
{1, 2, 3}	8.	Rbb	3, 6, 7 T
{1, 2, 3, 4}	9.	$Rbb \wedge \neg Rbb$	4, 8 T

11.

{1}	1.	$Pa \wedge Sa$	P
{2}	2.	$Qf(a) \wedge Raf(a)$	P
{3}	3.	$(\forall x)[Sx \rightarrow (\forall y)(Px \wedge Qy \rightarrow \neg Rxy)]$	P
{3}	4.	$Sa \rightarrow (\forall y)(Pa \wedge Qy \rightarrow \neg Ray)$	3 $US[a x]$
{1, 3}	5.	$(\forall y)(Pa \wedge Qy \rightarrow \neg Ray)$	1, 4 T
{1, 3}	6.	$Pa \wedge Qf(a) \rightarrow \neg Raf(a)$	5 $US[f(a) y]$
{1, 2, 3}	7.	$\neg Raf(a)$	1, 2, 6 T
{1, 2, 3}	8.	$Raf(a) \wedge \neg Raf(a)$	2, 7 T