MA 26600
FINAL EXAM INSTRUCTIONS
May 6, 2009

NAME ____________________ INSTRUCTOR ____________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in the instructor's name and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the SECTION NUMBER boxes with the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number, ask your instructor.)

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 23 questions, each worth an equal amount of points. Blacken in your choice of the correct answer in the spaces provided for questions 1–23. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

11. A table of Laplace Transforms can be found on the last page of the question sheets.

1. The general solutions to $ty' - y = t^2e^{-t}$ is

A. $y = -e^{-t} + c$

B. $y = -te^{-t}$

C. $y = -ce^{-t} + t$

D. $\boxed{y = -te^{-t} + ct}$

E. $y = te^{-t} + t$

2. The general solution of $\dfrac{dy}{dx} = \dfrac{3y - 2x}{y}$ is

A. $|y - x| = c|x|(y - 2x)^2$

B. $\boxed{|y - x| = c(y - 2x)^2}$

C. $(3y - 2x)^2 = c + |x|^{11}$

D. $\dfrac{y}{3x} + \dfrac{2}{9}\ln|\dfrac{3y}{x} - 2| = x + c$

E. $(3y - 2x)^2 = c + y$

3. A tank originally contains 100 gallons (gal) of water with a salt concentration of 1/2 lb/gal. A solution containing a salt concentration of 2 lb/gal enters at a rate of 2 gal/min. and the well-stirred mixture is pumped out at the rate of 1 gal/min. Then, the amount of salt in the tank after 50 min is

A. 0 lb

B. $400 - 350e^{-.5}$ lb

C. $\boxed{200\ lb}$

D. $-e^2$ lb

E. 100 lb

4. Which of the following contains all the asymptotically stable solution(s) for $y' = y(y-1)(y+2)$?

A. $y(t) = 1, -2$

B. $y(t) = -2$

C. $y(t) = 2$

D. $y(t) = 0, 1$

E. $\boxed{y(t) = 0}$

5. Which of the following is the implicit solution to the initial value problem

$$(e^x \sin y - 2y\sin x - 1) + (e^x \cos y + 2\cos x + 3)y' = 0, \; y(0) = \pi \; ?$$

A. $e^x \sin y + 2y\cos x + 3x - y = \pi$

B. $\boxed{e^x \sin y + 2y\cos x + 3y - x = 5\pi}$

C. $e^x \cos y - 2y\cos x - x + 3y = \pi$

D. $e^x \cos y + 2\sin x + 3x - y = -1 - \pi$

E. $e^x \sin y - 2y\cos x - x + 3y = \pi$

6. The **explicit** solution of the following initial value problem

$$y' = \frac{4x - 6x^2}{y}, \quad y(0) = -3$$

is

A. $y^2 = 4x^2 - 4x^3 + 9$

B. $\boxed{y = -\sqrt{4x^2 - 4x^3 + 9}}$

C. $\frac{y^2}{2} = 4x - 6x^2 + \frac{9}{2}$

D. $y = \sqrt{4x^2 - 4x^3 + 9}$

E. $y = -\sqrt{4x^2 - 6x^3 + 9}$

7. The function $y_1 = t$ is a solution of the differential equation

$$t^2y'' - ty' + y = 0.$$

Choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ forms a fundamental set of solutions to the differential equation.

A. $y_2 = t^3$

B. $y_2 = t \sin t$

C. $y_2 = t \cos t$

D. $\boxed{y_2 = t \ln t}$

E. $y_2 = te^t$

8. The proper form for the particular solution of the differential equation

$$y''' - 3y'' + 3y' - y = t \sin t + e^t$$

is

A. $At \sin t + Be^t$

B. $At + B + C \sin t + D \cos t + Ee^t$

C. $(At + B)(C \sin t) + Dt^3e^t$

D. $(At + B)(C \sin t + D \cos t) + Ete^t$

E. $\boxed{(At + B)(C \sin t + D \cos t) + Et^3e^t}$

9. The general solution of the differential equation

$$y''' - 2y'' + y' - 2y = 2e^t + 4$$

is

A. $\boxed{c_1 e^{2t} + c_2 \cos t + c_3 \sin t - e^t - 2}$

B. $e^{2t} + \cos t + \sin t - e^t + 2$

C. $c_1 e^t + c_2 \cos 2t + c_3 \sin 2t + e^t - 2$

D. $c_1 e^{2t} + c_2 \cos 2t + c_3 \sin 2t + e^t + 2$

E. $c_1 e^t + c_2 \cos t + c_3 \sin t$

10. Which of the following forms a fundamental set of solutions to the homogeneous differential equation $y^{(4)} + 8y'' + 16y = 0$?

A. $\{\cos 2t, \sin 2t, e^{2t}, e^{-2t}\}$

B. $\{e^{2t}, te^{2t}, e^{-2t}, te^{-2t}\}$

C. $\boxed{\{\cos 2t, t\sin 2t, t\cos 2t, \sin 2t\}}$

D. $\{e^{2t}, e^{-2t}\}$

E. $\{e^{2t}\cos 2t, e^{2t}\sin 2t, e^{-2t}\cos 2t, e^{-2t}\sin 2t\}$

11. A mass weighting 4 lb stretches a spring 1 ft. the mass is attached to viscous damper with a damping constant 2 lb-sec/ft. The mass is pulled down an additional 3 in, and then released. Let $u = u(t)$ denote the displacement of the mass from the equilibrium. (The gravity constant is $g = 32$ ft/sec^2.) Which of the following is satisfied by u?

A. $u'' + 16u' + 32u = 0,\ u(0) = 0,\ u'(0) = 0$

B. $\boxed{u'' + 16u' + 32u = 0,\ u(0) = 0.25,\ u'(0) = 0}$

C. $4u'' + u' + 32u = 0,\ u(0) = 0.25,\ u'(0) = 0$

D. $4u'' + u' + 2u = 0,\ u(0) = 0,\ u'(0) = 0$

E. $4u'' + 2u' + 2u = 0,\ u(0) = 0.25,\ u'(0) = 0$

12. One particular solution of the differential equation

$$y'' - 2y' + 2y = \frac{1}{\sin(t)}, \quad 0 < t < \pi$$

is

A. $\frac{1}{\sin(t)}$

B. $\boxed{\cos(t) + e^t \sin(t) \int e^{-t} \frac{\cos(t)}{\sin(t)}\, dt}$

C. $-\cos(t)e^t + e^t \sin(t) \int e^t \frac{\cos(t)}{\sin(t)}\, dt$

D. $-t\cos(t) + e^t \sin(t) \int \frac{\cos(t)}{\sin(t)}\, dt$

E. $\frac{1}{\cos(t)} + \frac{1}{\sin(t)}$

13. The inverse Laplace transform of

$$\frac{s^3+2}{s^4+s^2}$$

is

A. $t^2 + 2\cos t + \sin t$

B. $\cos t + 2\sin t$

C. $2\sin t - \cos t$

D. $\boxed{2t + \cos t - 2\sin t}$

E. $t + 1 + 2\sin t$

14. The Laplace transform of the function

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2t, & t \ge 1 \end{cases}$$

is

A. $s^{-2}(1 + e^{-s})$

B. $\boxed{s^{-2} + e^{-s}(s^{-2} + s^{-1})}$

C. $s^{-2}(1 + e^{-2s})$

D. $s^{-2}(1 + \frac{1}{4}e^{-s})$

E. $e^{-s}(s^{-1} + \frac{1}{4}s^{-2})$

15. Let $y(t)$ be the solution to the initial value problem

$$y'' + y = u_1(t), \quad y(0) = y'(0) = 0.$$

Then $y(\pi + 1)$ equals

A. -2

B. -1

C. 0

D. 1

E. $\boxed{2}$

16. The solution of the initial value problem

$$y'' + 6y' + 10y = \delta(t - 3), \quad y(0) = 1,\ y'(0) = -3$$

is

A. $u_3(t)e^{9-3t}\sin(t - 3)$

B. $\boxed{u_3(t)e^{9-3t}\sin(t - 3) + e^{-3t}\cos t}$

C. $u_3(t)e^{-3t}(\sin t + \cos t)$

D. $u_3(t)e^{-3t}\cos t$

E. $u_3(t)e^{3-3t}\cos(t - 3)$

17. The inverse Lapace transform of

$$\frac{1}{s^2(s-1)}$$

is

A. $e^t - 1$

B. te^t

C. $\boxed{\int_0^t (t-\tau)e^{\tau}\mathrm{d}\tau}$

D. $\int_0^\infty (t-\tau)e^{-\tau}\mathrm{d}\tau$

E. $\int_0^t (t-\tau)e^{-\tau}\mathrm{d}\tau$

18. In the phase portrait of the system $\vec{x}\,' = \begin{pmatrix} -1 & 1 \\ -4 & -1 \end{pmatrix} \vec{x}$, the origin is a

A. saddle point

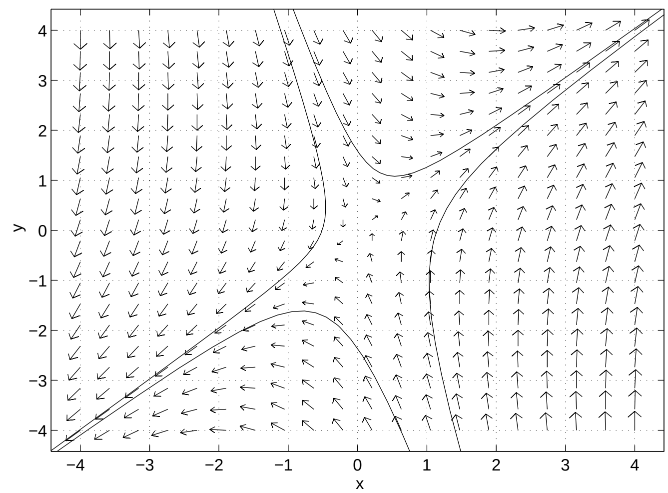
B. asymptotically stable node

C. asymptotically unstable node

D. $\boxed{\text{asymptotically stable spiral point}}$

E. asymptotically unstable spiral point

19. The phase portrait for a linear system of the form $\vec{x}\,' = \mathbf{A}\vec{x}$, where $\mathbf{A}$ is a 2×2 matrix, is shown below. If r_1 and r_2 denote the eigenvalues of $\mathbf{A}$, then what can you conclude about r_1 and r_2 by examining the phase portrait?



A. r_1 and r_2 are distinct and positive

B. r_1 and r_2 are distinct and negative

C. r_1 and r_2 have opposite signs

D. r_1 and r_2 are complex and have positive real part

E. r_1 and r_2 are complex and have negative real part

20. The general solution of the system $\vec{x}\,' = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \vec{x}$ is

A. $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t} \right]$

B. $c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + c_2 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} te^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t} \right]$

C. $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{t} \right]$

D. $c_1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} te^{t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{t} \right]$

E. $c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{t} \right]$

21. Let

$$\mathbf{A} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}, \ \vec{g} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

Which of the following is the general solution to the system $\vec{x}\,' = \mathbf{A}\vec{x} + e^{-2t}\vec{g}$?

A. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{-2t}$

B. $\boxed{c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 0 \\ -2 \end{pmatrix} e^{-2t}}$

C. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} e^{-2t}$

D. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 0 \\ -2 \end{pmatrix} e^{-2t}$

22. Consider the system $\vec{x}' = \begin{pmatrix} \alpha & 1 \\ -1 & 0 \end{pmatrix} \vec{x}.$

For what values of α is the origin an asymptotically unstable node?

A. $\alpha < -2$

B. $-2 < \alpha < 0$

C. $0 < \alpha < 2$

D. $\boxed{\alpha > 2}$

E. all real α

23. The function $x_1(t)$ determined by the initial value problem

$$\begin{cases} x_1' &= & & - & x_2 \\ x_2' &= & x_1 & & \end{cases}$$

with initial value $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $x_1(t) = \cos t + \sin t$

B. $\boxed{x_1(t) = \cos t - \sin t}$

C. $x_1(t) = 2\sin t + \cos t$

D. $x_1(t) = \frac{1}{2}(e^t + e^{-t})$

E. $x_1(t) = \cos t$