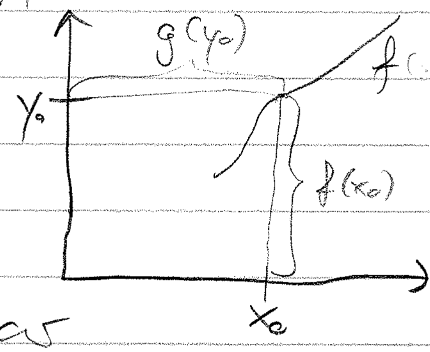
Case 2. $f'' = F(f, f')$

Let f be a solution and assume $f'(x_0) \neq 0$. for some x_0. Put $f(x_0) = y_0$. By the inverse function theorem $\exists$ function g near y_0 such that



$g(f(x)) = x$ for x near x_0

and $f(g(y)) = y$ for y near y_0. Put

$$v(y) = f'(g(y)).$$

Then $v(f(x)) = f'(x)$ so $f''(x) = \frac{d}{dx} f'(x) = v'(f(x)) f'(x)$

$\Rightarrow f''(g(y)) = v'(y) f'(g(y)) = v'(y) \cdot v(y)$

Hence our equation turns into

$$v'(y) \cdot v(y) = f''(g(y)) = F(f(g(y)), f'(g(y))) = F(y, v)$$

Assuming we can solve this 1st order eq, then f is given by

$$v(f(x)) = f'(x)$$

which is separable and can be solved.