Inductive Limits of C^*-algebras Related to Some Coverings

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In [5] E. G. Effros posed the problem of studying inductive limits of C^*-algebras of the form $C(X)\otimes M_n$. Because of the complexity of the possible $*$-homomorphisms $\Phi : C(X)\otimes M_n \to C(X)\otimes M_m$ (cf. [3] and [6]) it is reasonable to restrict our attention to specific classes of homomorphisms. In this paper we prove a unicity result concerning inductive limits associated with a sequence of coverings.

A unital homomorphism Φ is called homogeneous if for every $y\in Y$ the subalgebra $\Phi\big(C(X)\otimes 1_n\big)(y)\subset M_m$ has dimension $\frac{m}{n}$. (Note that n must divide m since Φ is unital.) Suppose that Y has the homotopy type of a finite CW-complex of dimension $\leq \frac{2m}{n}$ and that $K^\circ(Y)$ has no n-torsion. Then it follows from [3] that there is a $\left(\frac{m}{n}\right)$-fold covering $\psi : Z \to Y$, a monomorphism $\Phi_1 : C(Z)\otimes M_n \to C(Y)\otimes M_m$ which satisfies

$$(0)\qquad \Phi_1(g\circ\psi\otimes 1_n) = g\otimes 1_m, \qquad g\in C(Y),$$

and a continuous map $\varphi : Z\to X$ such that we have the factorization $\Phi = \Phi_1\circ\varphi^*$.

The homomorphism Φ_1 satisfying equation (0) are called compatible with the covering ψ or ψ-compatible, and they were introduced in [6] for other reasons. The previous decomposition confirms once more their importance, since they are now identified as the nontrivial part of the homogeneous homomorphisms.

We shall consider inductive limits with homomorphisms compatible with some appropriate coverings. Our result is based on a detailed description of such homomorphisms.

An interesting example is supplied by Bunce–Deddens algebras [2] which can be described as inductive limits of the form

$$(1)\qquad \dots \longrightarrow C(\mathbf{T}^m)\otimes M_{n_i} \longrightarrow C(\mathbf{T}^m)\otimes M_{n_{i+1}} \longrightarrow \dots$$

where $m=1$, $\mathbf{T}$ is the unit circle, and the homomorphisms Φ_i are compatible with the coverings $\mathbf{T}\ni z\mapsto z^{n_{i+1}/n_i}\in\mathbf{T}$.

In [6] C. Pasnicu has studied inductive limits of the form (1) with $m=2$ and he has proved that these limits do not depend on the particular choices

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of the homomorphisms Φ_i compatible with some product coverings $\mathbf{T}^2 \to \mathbf{T}^2$. Moreover, these limits were seen to be isomorphic to tensor products of two Bunce–Deddens algebras.

The aim of this paper is to consider the same problem in an abstract setting.

Given a free action of $\mathbf{T}^m$ on a compact connected manifold X and a strictly increasing sequence of finite subgroups of $\mathbf{T}^m$:

$$G_1 \subset G_2 \subset \ldots \subset G_i \subset G_{i+1} \subset \ldots \subset \mathbf{T}^m,$$

we consider inductive limits of the form

$$L = \varinjlim\left(\to C(X_i) \otimes M_{n_i} \xrightarrow{\Phi_i} C(X_{i+1}) \otimes M_{n_{i+1}} \longrightarrow\right),$$

where $X_i = \frac{X}{G_i}$, $n_i = |G_i|$, and the homomorphisms Φ_i are compatible with the coverings $X_i \to X_{i+1}$.

Under some topological restrictions involving the absence of torsion in the cohomology $H^*(X_i, \mathbf{Z})$, we prove that the inductive limit L does not depend on Φ_i and it is isomorphic to the C^*–algebra transformation group $C(X) \rtimes G$, where

$$G = \bigcup_{i=1}^{\infty} G_i.$$

For the case of Bunce–Deddens algebras this isomorphism was noticed by P. Green.

As a corollary, we extend the result from [6] to the m–dimensional torus.

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1. Preliminaries. We shall denote by M_n the C^*–algebra of $n \times n$ complex matrices and by 1_n its unit.

Suppose that X is a compact, connected, real manifold and let S be a finite group acting freely on X.

If $k = |S|$ (the order of S), then the quotient map onto the orbit space $\psi : X \to \frac{X}{S}$ is a regular k–fold covering. Let n be a positive integer.

We recall from [6] that a unital homomorphism

$$\Phi : C(X) \otimes M_n \to C\left(\tfrac{X}{S}\right) \otimes M_{kn}$$

is called compatible with the covering ψ or ψ–compatible if

$$\Phi(g \circ \psi \otimes 1_n) = g \otimes 1_{kn}, \qquad g \in C\left(\tfrac{X}{S}\right). \tag{2}$$

Looking at the following diagram

$$\begin{array}{ccccc} C(X) \otimes M_n & & \xrightarrow{\Phi} & & C\left(\frac{X}{S}\right) \otimes M_{kn} \\ & \nwarrow^{\psi^*} & & \nearrow_{\alpha} & \\ & & C\left(\frac{X}{S}\right) & & \end{array}$$

where $\psi^*(g) = (g \circ \psi) \otimes 1_n$ and $\alpha(g) = g \otimes 1_{kn}$, it is clear that ψ–compatible homomorphisms may be viewed as a kind of section for the fibering $X \to \frac{X}{S}$.

Assume that the K–theory group $K^\circ(X)$ is torsion–free and that $\dim(X) \leq 2k$. Then it follows from [3, Theorem 1.3] that

$$\Phi = v(\Phi' \otimes \mathrm{id}_{M_n})v^*$$

for some unitary $v \in C(\frac{X}{S}) \otimes M_{kn}$ and some ψ–compatible homomorphism $\Phi' : C(X) \to C(\frac{X}{S}) \otimes M_k$. Moreover, it is proved in [3] that there is a continuous map $p : X \to P(\mathbf{C}^k) =$ the space of all one dimensional self–adjoint projections acting on $\mathbf{C}^k$, such that Φ' is given by the following formula:

$$\Phi'(f)(\psi(x)) = \sum_{s\in S} f(s\cdot x)p(s\cdot x), \qquad f \in C(X),\, x \in X. \tag{3}$$

Of course since Φ' is unital we must have

$$\sum_{s\in S} p(s\cdot x) = 1_n, \qquad x \in X. \tag{4}$$

Also, it is clear that both Φ and Φ' are monomorphisms. Despite the previous description, we don't know a priori if ψ–compatible homomorphisms (or equivalently maps $p : X \to P(\mathbf{C}^k)$ satisfying (4)) do exist.

However, if we assume that S is an abelian group and that the second cohomology group $H^2(\frac{X}{S}, \mathbf{Z})$ is torsion–free, then such homomorphisms can be constructed as follows. By [3] there is a continuous map

$$u : X \to U(k) \simeq U(\ell^2(S))$$

such that $u(s\cdot x) = \rho(s)u(x)$, $x \in X$, $s \in S$, where $\rho : S \to U(k)$ is the right regular representation of S. Now if $\langle e_s^\circ\rangle_{s\in S}$ are the orthogonal projections onto the subspaces $[\delta_s]$ spanned by the vectors in the canonical basis $\langle\delta_s\rangle_{s\in S}$ of $\ell^2(S)$, then the homomorphism $\Phi' : C(X) \to C(\frac{X}{S}) \otimes M_k$ given by

$$\Phi'(f)(\psi(x)) = u(x)^*\big(\sum_{s\in S} f(s\cdot x)e_s^\circ\big)u(x), \qquad f \in C(X),\, x \in X \tag{5}$$

is compatible with the covering $X \to \frac{X}{S}$. Note that Φ' is well defined since $\rho(s)^* e_t^\circ \rho(s) = e_{ts}^\circ$, t, $s \in S$. Consider now the crossed–product C^*–algebra:

$$C(X) \rtimes S = \langle F \in C(X) \otimes M_k : f(s\cdot x) = \rho(s)F(x)\rho(s)^*,\, x \in X,\, s \in S\rangle.$$

Then the unitary u can be used to give an isomorphism $H : C(X) \rtimes S \to C(\frac{X}{S}) \otimes M_k$. To see this, we identify $C(\frac{X}{S}) \otimes M_k$ with

$$\big\langle F \in C(X) \otimes M_k : F(s\cdot x) = F(x) \qquad \text{for all } x \in X,\, s \in S\big\rangle,$$

and we take $H(F) = u^*Fu$. Note that if

$$j : C(X) \to C(X) \rtimes S, \qquad j(f)(x) = \sum_{s\in S} f(s\cdot x)e_s^\circ,$$

is the canonical embedding, then the isomorphism H is such that $H \circ j = \varphi'$.

2. Inner Equivalence. Assume $S \subset \mathbf{T}^m$ and also that the action of S on X is induced by a continuous free action of $\mathbf{T}^m$ on X.

Then we are able to give a more complete description of homomorphisms Φ which are compatible with the covering $X \to \frac{X}{S}$. Our description will imply that any two such homomorphisms are inner equivalent.

Lemma 2.1. *Let $x_0 \in X$, let $p : X \to P(\mathbf{C}^k)$ be a continuous map which satisfies equation (4) and assume that $H^2(X,\mathbf{Z})$ is torsion-free. Then there is a continuous map $u : X \to U(k)$ such that*

$$p(s \cdot x) = u(x)^* p(s \cdot x_0) u(x), \qquad x \in X,\, s \in S. \tag{6}$$

Proof. Set $e_s(x) = p(s \cdot x)$. Then $\langle e_s \rangle_{s \in S}$ is a partition of the unity in the C^*-algebra $C(X) \otimes M_k$. Since the action of S on X is induced by a continuous action of $\mathbf{T}^m$ which is a pathwise connected space, it follows that the projections $\langle e_s \rangle_{s \in S}$ are mutually equivalent in $C(X) \otimes M_k$. To see this let $a : [0,1] \to \mathbf{T}^m$ be a continuous path from 1 to s. Then $e_{a(t)}(x) = p\big(a(t) \cdot x\big)$, $t \in [0,1]$ is a continuous path of projections from e_1 to e_s. By a standard argument we find now a partial isometry $e_{s,1} \in C(X) \otimes M_k$ such that $e_{s,1}^* e_{s,1} = e_1$ and $e_{s,1} e_{s,1}^* = e_s$. Define $e_{s,s} = e_s$ and $e_{s,t} = e_{s,1} e_{t,1}^*$ to obtain a system of matrix units in $C(X) \otimes M_k$. Now consider the C^*-homomorphisms Φ, $\Phi_0 : M_k \to C(X) \otimes M_k$ given by

$$\Phi_0\big(e_{s,t}(x_0)\big) = 1_{C(X)} \otimes e_{s,t}(x_0)$$

$$\Phi\big(e_{s,t}(x_0)\big) = e_{s,t}.$$

(Note that in the definition above we identified M_k with the C^*-algebra generated by $\langle e_{s,t}(x_0) \rangle_{s,t \in S}$). Since the complex line bundles on X are classified by $H^2(X,\mathbf{Z})$, which we suppose to be torsion-free, it follows from [3, Prop. 1.1] that there is some unitary $u \in C(X) \otimes M_k$ such that $\Phi_0 = u \Phi u^*$. This implies that

$$e_s(x_0) = u(x) e_s(x) u(x)^*.$$

Hence

$$p(s \cdot x) = u(x)^* p(s \cdot x_0) u(x)$$

for all x in X and $s \in S$. □

Let $\rho : S \to B\big(\ell^2(S)\big)$ be the regular representation of S. We identify $B\big(\ell^2(S)\big)$ with $C^*\langle e_{s,t}(x_0) : s,\, t \in S \rangle \simeq M_k$, so that $\rho(r)^* e^\circ_{s,t} \rho(r) = e^\circ_{sr,tr}$. As an easy consequence of the equation (6) we obtain that every $u(s \cdot x) u(x)^* \rho(s)^*$ commutes with all $e_{t,t}(x_0)$, $t \in S$. Setting $w_s(x) = u(s \cdot x) u(x)^* \rho(s)^*$, it follows that w_s is diagonal with respect to the projections $e_{t,t}(x_0)$. More precisely, there are continuous functions $w(t,s) : X \to \mathbf{T}$, $s,t \in S$, such that

$$w_s(x) = \sum_{t \in S} w_{t,s}(x) e_{t,t}(x_0), \qquad x \in X. \tag{7}$$

Moreover, it follows from the definition of $(w_s)_{s\in S}$ that

$$w_s(t\cdot x) = w_{st}(x)\rho(s)w_t(x)^*\rho(s)^* .$$

Then we have corresponding relations for $w_{t,s}$:

$$w_{t,sr}(x) = w_{ts,r}(x)w_{t,s}(r\cdot x), \qquad r,\, s,\, t\in S,\, x\in X. \tag{8}$$

Equation (8) looks like some "cocycle relations." Our next task is to resolve the "cocycle $(w_{s,t})$," i.e., to find continuous maps $d_s : X \to \mathbf{T}$, $s\in S$, such that

$$w_{t,s}(x) = d_{ts}(x)d_t(s\cdot x)^{-1}, \qquad x\in X,\, s,t\in S. \tag{9}$$

Suppose now that the maps (d_s) have been found and set

$$v(x) = \Big(\sum_{s\in S} d_s(x)e_{s,s}(x_0)\Big)u(x).$$

Then an easy computation shows us that

$$v(s\cdot x) = \rho(s)v(x), \qquad x\in X,\, s\in S, \text{ and} \tag{10}$$

$$p(x) = v(x)^*p(x_0)v(x), \qquad x\in X. \tag{11}$$

It is rather surprising that the cocyle $(w_{s,t})$ can be resolved very easily. For instance, we can choose $d_t(x) = w_{1,t}(x)$. Thus we arrive at the following theorem.

Theorem 2.2. *Assume that $H^2(X,\mathbf{Z})$ is torsion free. If $\Phi : C(X) \to C\big(\frac{X}{S}\big)\otimes M_k = C\big(\frac{X}{S}\big)\otimes B\big(\ell^2(S)\big)$ is a $*$-homomorphism compatible with the covering $\psi : X \to \frac{X}{S}$, then there is some unitary valued map $v : X \to U(k) = U\big(\ell^2(S)\big)$ such that*

$$v(s\cdot x) = \rho(s)v(x), \qquad x\in X,\, s\in S, \tag{12}$$

$$\Phi(f)\big(\psi(x)\big) = v(x)^*\Big(\sum_{s\in S} f(s\cdot x)e_0^\circ\Big)v(x). \tag{13}$$

Proof. For any unital ψ-compatible homomorphism we have the description provided by equation (3). Using Lemma 2.1, we find, by the previous discussion, that

$$p(s\cdot x) = v(x)^*p(s\cdot \dot{x}_0)v(x) = v(x)^*e_s(x_0)v(x)$$

with v satisfying equation (10). □

Corollary 2.3. *Assume that $H^2(X,\mathbf{Z})$ is torsion free. Then any two homomorphisms $\Phi,\Psi : C(X)\to C\big(\frac{X}{S}\big)\otimes M_k$ compatible with the covering $X\to \frac{X}{S}$ are inner equivalent, that is, $\Psi = u\Phi u^*$ for some unitary $u\in C\big(\frac{X}{S}\big)\otimes M_k$.*

Proof. Theorem 2.1 provides descriptions of Φ and Ψ with appropriate unitaries v and w. After conjugating with a unitary in $C\left(\frac{X}{S}\right) \otimes M_k$, we may assume that these descriptions are given relative to the same projections (e_s°). Consequently, we may choose $u = w^* v$ since

$$u(s \cdot x) = w(s \cdot x)^* v(s \cdot x) = w(x)^* \rho(s)^* \rho(s) v(x) = u(x)$$

and it is clear from (13) that $\Psi = u \Phi u^*$. □

Example. Let $\mathbf{S}^2$ be the two-sphere and let

$$\mathbf{P}^2 = \frac{\mathbf{S}^2}{\mathbf{Z}_2}$$

be the two-dimensional real projective space. Since the action of $\mathbf{Z}_2$ on $\mathbf{S}^2$ is not induced by the continuous actions of some connected Lie group including $\mathbf{Z}_2$, Corollary 2.3 does not apply. In fact we show that there are infinitely many homomorphisms $\Phi_k : C(\mathbf{S}^2) \to C(\mathbf{P}^2) \otimes M_2$ compatible with the canonical covering $\psi : \mathbf{S}^2 \to \mathbf{P}^2$ which are not inner equivalent. Let $\alpha : \mathbf{S}^2 \to \widetilde{\mathbf{C}}$ be the stereographic projection, and let $\beta : \mathbf{S}^2 \to P(\mathbf{C}^2)$ be the canonical homeomorphism that sends antipodal points to orthogonal projections. If $g_k : \widetilde{\mathbf{C}} \to \widetilde{\mathbf{C}}$ is given by $g_k(z) = z^{2k+1}$, then $q_k : \mathbf{S}^2 \to \mathbf{S}^2$, $q_k = \alpha^{-1} g_k \alpha$, satisfies $q_k(-x) = -q_k(x)$ for any $x \in \mathbf{S}^2$. Therefore $p_k = \beta q_k$ satisfies equation (4) so that the formula

$$\Phi_k(f)\big(\psi(x)\big) = f(x) p_k(x) + f(-x) p_k(-x)$$

defines a ψ-compatible homomorphism from $C(\mathbf{S}^2)$ to $C(\mathbf{P}^2) \otimes M_2$. Assuming that Φ_k and Φ_r are inner equivalent, we get a unitary $u \in C(\mathbf{P}^2) \otimes M_2$ such that $p_k = u p_r u^*$. Since the homotopy class $[\mathbf{P}^2, U(2)] = [\mathbf{P}^2, \mathbf{S}^1 \times \mathbf{S}^3]$ reduces to the null element, it follows that p_k and p_r are homotopic. Therefore g_k and g_r are homotopic, and this implies that $k = r$.

3. Some inductive limits. As in the previous section, we start with a continuous free action of $\mathbf{T}^m$ on a compact, connected, real manifold X. Let

$$G_1 \subset G_2 \subset \ldots \subset G_i \subset G_{i+1} \ldots$$

be an infinite tower of finite subgroups of $\mathbf{T}^m$. Let

$$n_i = |G_i|, \qquad k_i = \left|\frac{G_{i+1}}{G_i}\right|,$$

and note that $n_{i+1} = n_i k_i$. If X_i denotes the quotient space $\frac{X}{G_i}$, we have a natural k_i-fold covering $X_i \to X_{i+1}$ whose deck-group S_i is isomorphic to $\frac{G_{i+1}}{G_i}$.

In this section we deal with inductive limits of the form

$$(14) \qquad \ldots \to C(X_i) \otimes M_{n_i} \xrightarrow{\Phi_i} C(X_{i+1}) \otimes M_{n_{i+1}} \to \ldots,$$

where each homomorphism Φ_i is compatible with the covering $X_i \to X_{i+1}$.

The main result is the following:

Theorem 3.1. *Assume that the manifolds X_i have no torsion in cohomology, i.e., $H^*(X_i, \mathbf{Z})$ is torsion-free for any $i \geq 1$.*

Then the inductive limit $\big(C(X_i) \otimes M_{n_i}, \Phi_i\big)$ does not depend on the particular choice of the homomorphisms Φ_i. In fact, it depends only on the group $G = \bigcup_{i=1}^{\infty} G_i$ since it is isomorphic to the crossed product C^*-algebra $C(X) \rtimes G$.

Proof. By considering a refinement of the sequence in (14), we may assume that $\dim X_i \leq \frac{n_{i+1}}{n_i}$. As a first step, we prove that any two homomorphisms Φ_i, $\Psi_i : C(X_i) \otimes M_{n_i} \to C(X_{i+1}) \otimes M_{n_{i+1}}$ are inner equivalent. Recall that $n_{i+1} = n_i k_i$. Since $H^*(X_{i+1}, \mathbf{Z})$ is torsion free, it follows from [1] that $K^{\circ}(X_{i+1})$ is torsion free. Hence, by the results quoted in Section 1, we may assume that $n_i = 1$. At this point the assertion follows from Corollary 2.7. To conclude the first part of the theorem, we recall Lemma 2.1 of [5], which asserts that the inductive limits $\lim(A_i, \Phi_i)$ and $\lim(A_i, \Psi_i)$ are isomorphic if the homomorphisms Φ_i and Ψ_i are inner equivalent.

To proceed further, let us consider the diagram

$$\begin{array}{ccccccc} \cdots \longrightarrow & C(X) \rtimes G_i & \xrightarrow{J_i} & C(X) \rtimes G_{i+1} & \longrightarrow \cdots \\ & \Big\downarrow H_i & & \Big\downarrow H_{i+1} & \\ \cdots \longrightarrow & C(X_i) \otimes M_{n_i} & \xrightarrow{\Phi_i} & C(X_i) \otimes M_{n_{i+1}} & \longrightarrow \cdots \end{array} \tag{15}$$

where (J_i) are the canonical embeddings, (H_i) are the isomorphisms described in Section 1, and (Φ_i) are chosen such that $\Phi_i = H_{i+1} J_i H_i^{-1}$. With this definition it is straightforward to check that the homomorphisms Φ_i are compatible with the coverings $X_i \to X_{i+1}$. Since the inductive limit of the upper row in the diagram (15) is equal to

$$C(X) \rtimes G = \Big(\bigcup_{i=1}^{\infty} C(X) \rtimes G_i\Big)^{-},$$

it turns out that the unique limit that arises from the diagram (14) is isomorphic to $C(X) \rtimes G$. □

Let $\mathbf{T}^m$ act on $X = \mathbf{T}^m$ by translations. Given a finite subgroup S of $\mathbf{T}^m$, it is well known that

$$\frac{\mathbf{T}^m}{S} \simeq \mathbf{T}^m.$$

Further, since $H^*(\mathbf{T}, \mathbf{Z}) = \mathbf{Z} \oplus \mathbf{Z}$, it follows by Künneth's formula that $H^*(\mathbf{T}^m, \mathbf{Z})$ is torsion free. Therefore we may apply Theorem 3.1 to obtain a unicity result concerning the inductive limits on the form

$$\cdots \to C(\mathbf{T}^m) \otimes M_{n_i} \xrightarrow{\Phi_i} C(\mathbf{T}^m) \otimes M_{n_{i+1}} \to \cdots, \tag{16}$$

where the homomorphisms Φ_i are compatible with $\left(\frac{n_{i+1}}{n_i}\right)$–fold coverings $\mathbf{T}^m \to \mathbf{T}^m$. Moreover, if these coverings correspond to the tower of subgroups

$$G_1 \subset G_2 \subset \ldots \subset \mathbf{T}^m,$$

and we assume that $G = \bigcup_{i=1}^{\infty} G_i$ is dense in $\mathbf{T}^m$, then it can be proved that the C^*–algebra $C(\mathbf{T}^m) \rtimes G$ is simple and it has a unique faithful trace state.

Suppose now that the homomorphisms Φ_i are compatible with the coverings

$$(17) \qquad (\mathbf{z}_1, \ldots, \mathbf{z}_m) \mapsto \left(\mathbf{z}_1^{p_1(i)}, \ldots, \mathbf{z}_m^{p_m(i)}\right),$$

and let $n_k(i) = \prod_{j=1}^{i} P_k(j)$, $1 \le k \le m$. Let $A(n_k)$ be the Bunce–Deddens algebra associated with the generalized integer $n_k = \left(n_k(i)\right)_{i \ge 1}$. Then we have the following corollary, which extends the main result of [6].

Corollary 3.2. *The inductive limit* (16) *does not depend on the choice of the homomorphisms* Φ_i *compatible with the coverings* (17). *Moreover, it is isomorphic to the* C^**-tensor product* $\bigotimes_{k=1}^{m} A(n_k)$.

Proof. We apply Theorem 3.1 with

$$G_i = G_1(i) \times G_2(i) \times \cdots \times G_m(i),$$

where

$$G_k(i) = \langle z \in \mathbf{T} : \mathbf{z}^{n_k(i)} = 1 \rangle.$$

Let $G_k = \bigcup_{i=1}^{\infty} G_k(i)$ and note that $G = G_1 \times G_2 \times \cdots \times G_m$. If we denote by L the unique limit arising from (16), then

$$L = C(\mathbf{T}^m) \rtimes G \simeq \bigotimes_{k=1}^{m} C(\mathbf{T}) \rtimes G_k$$

and

$$C(\mathbf{T}) \rtimes G_k \simeq A(n_k). \qquad \square$$

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