

# WABASH EXTRAMURAL MODERN ANALYSIS SEMINAR

April 12

2:00 p.m.

at

## Wabash College

in rooms 114 and 118 Baxter Hall

*Times given are Eastern Daylight Time,  
which is currently local time for Central Indiana and Ohio.*

- 2:00–2:30     *Refreshments and conversation*
- 2:30–3:30     Commutants of Finite Blaschke Product Multiplication Operators on Bergman Spaces  
*CARL C. COWEN, IUPUI*
- 3:30–4:00     *More refreshments and conversation*
- 4:00–5:00     Unitary representations of polycyclic groups  
*CALEB ECKHARDT, Miami University*
- 5:00–...     *Refreshments and farewells*

The purpose of Wabash Seminar talks is to present surveys of interest to all analysts, including graduate students and scholars working in areas far from the speaker's specialty. Come and meet your fellow analysts, learn what's going on, and spread the word.

Next Meeting: TBA

*For further information call*

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# Commutants of Finite Blaschke Product Multiplication Operators on Bergman Spaces

CARL C. COWEN

Except in special circumstances, it is quite difficult to determine conditions that characterize which operators commute with a given operator. Such special circumstances include self-adjoint and normal operators (where the spectral theorem can be used) and cases in which the operator in question has a rich point spectrum. The results in this latter situation often come from the application of the easy observation that if  $A$  and  $B$  commute, the eigenspaces of  $A$  are invariant for  $B$ .

If  $\mathcal{H}$  is a Hilbert space of analytic functions on the unit disk and  $T_z$  is the operator of multiplication by  $z$ , it is well known that the commutant of  $T_z$  is the collection of multiplication operators  $T_f$  where  $f$  is a bounded analytic function on the disk,  $f$  is in multiplier algebra for  $\mathcal{H}$ , and  $(T_f h)(z) = f(z)h(z)$ .

In the 1970's and 80's, the question "Which operators on the Hardy space  $H^2(\mathbb{D})$  commute with  $T_f$  for  $f$  a bounded analytic function on the disk?" was investigated. More recently, there has been interest in this question for the Bergman space  $A^2(\mathbb{D})$  and weighted Bergman spaces. In this talk, an overview of the work of thirty years ago will be presented and we will consider this question for  $f = B$ , a finite Blaschke product, for  $T_B$  acting on a broad collection of spaces containing  $H^2(\mathbb{D})$ , a question that has wider consequences than might be expected. In particular, we show that the commutants of the operators  $T_B$  are the same on all of these spaces!

This is joint work with Rebecca G. Wahl at Butler University.

## Unitary representations of polycyclic groups

CALEB ECKHARDT

If  $\Gamma$  is a non-Type I discrete group, there is essentially no hope of a reasonable characterization of its irreducible representations up to unitary equivalence. If one instead focuses on only the  $C^*$ -algebras generated by irreducible representations of  $\Gamma$  the situation is a little more promising. Recent events in the theory of  $C^*$ -algebras suggest that—in the case  $\Gamma$  is a finitely generated torsion free nilpotent group—ordered  $K$ -theory may serve as a complete invariant for the  $C^*$ -algebras generated by irreducible representations of  $\Gamma$ . In this talk we will discuss quasi-diagonality of unitary representations of polycyclic groups (all finitely generated nilpotent groups are polycyclic) and how this feeds into the larger goal of characterizing  $C^*$ -algebras generated by irreducible representation of nilpotent groups. Some of this is joint work with Craig Kleski and Paul McKenney.