WABASH MINI-CONFERENCE, September 21-22, 2019

Titles and Abstracts

PLENARY TALKS

KONRAD AGUILAR, Arizona State University

Fell Topologies for AF Algebras and the Quantum Propinquity

Abstract: We introduce a topology on the ideal space of any C*-inductive limit built by an inverse limit of topologies from the ideal spaces of the inductive sequence and produce conditions for when this topology agrees with the Fell topology. With this topology, we find that convergence of certain ideals of an AF algebra can provide convergence of their quotients in the quantum Gromov-Hausdorff propinquity building from previous joint work with Latremoliere. This bestows a continuous map from a class of ideals of the Boca-Mundici AF algebra equipped with either the Jacobson or Fell topologies, to the space of quotients equipped with the propinquity topology.

ADAM DOR-ON, University of Illinois Urbana-Champaign and University of Copenhagen

Classification of irreversible and reversible operator algebras

Abstract: C*-algebras have been intensely studied in recent years, especially through the lens of K-theory, classification and the Eilliott program. Prominent advances in non-simple classification include an abundance of results for Cuntz-Krieger algebras of a directed graph. One such result of Cuntz and Krieger shows that the extension groups of such algebras coincide with Bowen-Franks groups of the subshift of finite type associated to the graph. A more recent achievement due to Eilers, Restorff, Ruiz and Sorensen is the complete classification of Cuntz-Krieger algebras of finite graphs up to a prescribed set of deformations of the graph arising from the subshift.

On the other hand, classifying non-self-adjoint algebras is an effort initiated by Arveson and Josephson in their late 60s paper on algebras arising from measure preserving dynamics. This was later taken up by many authors, including Davidson and Katsoulis who classified non-self-adjoint algebras arising from dynamical systems on topological spaces. In the late 90s, Muhly and Solel established a vastly applicable non-commutative function theory where completely contractive representations of tensor and Hardy algebras are treated as points. This framework allows us to treat many non-self-adjoint classification problems by modeling the algebras as tensor or Hardy algebras of C*-correspondences.

In this talk we will connect these seemingly unrelated topics, survey some pertinent results from the literature and uncover a striking hierarchy of classification for irreversible and reversible operator algebras.

DAVID KERR, Texas A&M University

Dynamical alternating groups, property Gamma, and inner amenability

Abstract: I will show that the alternating group of a topologically free action of a countably infinite amenable group on the Cantor set has property Gamma (and in particular is inner amenable) and that there are large classes of such groups which are simple, finitely generated, and nonamenable. This is joint work with Robin Tucker-Drob.

BRENT NELSON, Michigan State University

Free Stein Irregularity and Dimension

Abstract: Given an n-tuple X of non-commutative random variables, its free Stein discrepancy relative to the semicircle law (the non-commutative analogue of classical Stein discrepancy relative to the Gaussian distribution) measures how "close" the distribution of X is to the semicircle law. By considering free Stein discrepancies relative to a broader class of laws, one can define a quantity called the free Stein irregularity. I will discuss this quantity and show how it can be related to other free probabilistic quantities such as the free Fisher information and the non-microstates free entropy dimension. I will also show how it can be easily computed for a number of interesting examples. This is based on joint work with Ian Charlesworth.

GELU POPESCU, University of Texas San Antonio

Multi-Toeplitz operators associated with noncommutative domains

Abstract: We present recent results concerning multi-Toeplitz operators associated with noncommutative domains $\mathbf{D}_q^m(\mathcal{H}) \subset B(\mathcal{H})^n$, $m, n \geq 1$, where $B(\mathcal{H})$ is the algebra of all bounded linear operators on a Hilbert space $\mathcal{H}$. These operators are acting on the full Fock space with n generators and have as symbols free pluriharmonic functions on the interior $\mathbf{D}_q^m(\mathcal{H})^\circ$ of the domain $\mathbf{D}_q^m(\mathcal{H})$. One of the main results shows the set of all multi-Toeplitz operators coincides with

$$\overline{\mathcal{A}(\mathbf{D}_q^m)^* + \mathcal{A}(\mathbf{D}_q^m)}^{WOT},$$

where the domain algebra $\mathcal{A}(\mathbf{D}_q^m)$ is the norm-closed unital non-selfadjoint algebra generated by the universal model $(W_1, \ldots, W_n)$ of the noncommutative domain $\mathbf{D}_q^m(\mathcal{H})$. These results are used to study the class of free pluriharmonic functions on $\mathbf{D}_q^m(\mathcal{H})^\circ$. Several classical results from complex analysis concerning harmonic functions have analogues in our noncommutative setting. In particular, we show that the bounded free pluriharmonic functions are precisely those which are noncommutative Berezin transforms of multi-Toeplitz operators, and solve the Dirichlet extension problem in this setting.

We also present a Brown-Halmos characterization of the multi-Toeplitz operators associated with the noncommutative m-hyperball (the case when $q = Z_1 + \cdots + Z_n, m \geq 2$), which is a noncommutative version of Eschmeier and Langendörfer recent commutative result. Our result shows that the multi-Toeplitz operators are characterized by an algebraic equation involving the universal model $(W_1, \ldots, W_n)$ of the noncommutative m-hyperball.

HANG WANG, East China Normal University

A K-theoretic Selberg trace formula

Abstract: The close relationship between index theory and representation theory is a classical theme. In particular, the trace formula has been studied through the lens of index theory by several researchers already. In the joint work with Bram Mesland and Haluk Sengun, we take this connection further and obtain a formulation of the trace formula in K-theoretic terms. The central objects here are the K-theory groups of the C^*-algebras associated to a semisimple Lie group G and a uniform lattice Γ. The geometric side of the K-theoretic trace formula is calculated by the equivariant indices of some Dirac type operators and the spectral side is a finite sum involving multiplicities of unitary representations in $L^2(G/\Gamma)$. This work is part of a program which explores the potential role that operator K-theory could play in the theory of automorphic forms.

SESSION TALKS

ROY ARAIZA, Purdue University:

Matrix Convex Sets, Tensor Products, and Non-Commutative Choquet Boundaries

Abstract: Non-commutative convexity has been studied as early as the 1980s beginning with the work of Wittstock and later looked at by Effros, Webster and Winkler. Non-commutative Choquet theory was studied in detail in a series of papers by Arveson as recent as the early 2000s. Building on the tensor theory of operator systems developed by Kavruk et al. I will discuss recent work in developing a theory of tensor products of compact matrix convex sets and discuss when such tensor products can be realized as the closed matrix convex hull of their Choquet points. This is joint work with Adam Dor-On and Thomas Sinclair.

LANGWEN HUI and YIFAN ZHANG, University of Illinois Urbana-Champaign:

Conditional Equivalence and Non-commutative Peaking of Markov Chains

Abstract: Given a Markov chain P, we can construct a (non-self-adjoint) operator algebra $\mathcal{T}_+(P)$ based on conditional probabilities. Using the characterization of Dor-On and Markiewicz we show that any two distinct strongly Liouville Markov chains still produce non-isomorphic algebras. This is accomplished by using Doob transforms to characterize when a Markov chain has strong Liouville property in terms of conditional probabilities. For Markov chains with finite state space, Dor-On and Markiewicz also proved a sufficient condition for a state to be in the "boundary", the set of states essential for calculating the complete norm of operators in $\mathcal{T}_+(P)$. We improve this to a sufficient and necessary characterization of the boundary. This is joint work with Xinxin Chen, Adam Dor-On, and Christopher Linden.

NICHOLAS LARACUENTE, University of Illinois Urbana-Champaign:

Decay Estimates for Quantum Markov Semigroups

Abstract: We study decay of relative entropy and trace distance from the fixpoint algebras of particular classes of Quantum Markov Semigroups (QMSs). A particularly strong kind of decay bound is a complete logarithmic Sobolev inequality (CLSI), which is tensor-stable and implies immediate decay of relative entropy. We show a variety of decay estimates for QMSs described by ergodic, finite transition graphs. Our decay estimates have potential applications in quantum information. This includes joint work with Li Gao and Marius Junge.

HAOJIAN LI, University of Illinois Urbana-Champaign:

Complete Logarithmic Sobolev Inequality

Abstract: On a weighted Riemannian manifold $(M, g, e^{-u}dvol)$, the Bakry-Emery tensor is defined by $Rc_\infty = Rc + Hess_u$. Thanks to the celebrated Bakry-Emery theorem, $\mu = e^{-u}$ satisfies the ρ-logarithmic Sobolev inequalities provided $Rc_\infty \geq \rho > 0$. We show that it also serves as the criterion of complete logarithmic Sobolev inequality (CLSI), and that CLSI is stable under bounded perturbation. We can also extend our machinery to learn a family of functions which satisfy complete Sobolev type inequality (CFSI). (Joint work with Marius Junge)

CHRISTOPHER LINDEN, University of Illinois Urbana-Champaign:

Self-adjoint free semigroupoid algebras for graphs

Abstract: By work of Glimm, the representation theory of the Cuntz algebras is intractable. By weakening the notion of equivalence and considering isomorphism classes of not-necessarily self-adjoint free semigroup algebras, it is still possible to distinguish many representations. For Toeplitz-Cuntz-Krieger representations of directed graphs, Davidson, Dor-On, and Li considered free semigroupoid algebras. A construction of Read shows that $\mathcal{O}_2$ admits a free semigroup algebra which is actually self-adjoint. Davidson, Dor-On, and Li gave necessary and sufficient conditions for a finite directed graph to admit such a self-adjoint free semigroup algebra. In this talk I will discuss recent joint work with Adam Dor-On, in which we use apply the periodic road coloring theorem of Béal and Perrin to establish the converse of this necessary condition. A finite directed graph admits a self-adjoint free semigroupoid algebra if and only if it is the union of transitive components.

GARRETT MULCAHY, Purdue University:

Approximative Commutation of Matrices

Abstract: John von Neumann's "Approximative Properties of Matrices of High Finite Order"(1941) explores the asymptotic behavior of matrices as dimension increases but remains finite. Essentially, von Neumann ventured to explore the neglected middle ground between finite and infinite dimensional analysis. The major result of this paper is a proof of the existence of "big, bad matrices"— that is, matrices of large dimension that possess "bad" qualities. von Neumann's proof was nonconstructive, making use of what he called a "volumetric" argument. We utilize computa-

tional techniques in a quest to find a construction of these matrices; discovering what the matrices look like will potentially have applications to data science and the theory of random matrices.

MEREDITH SARGENT, University of Arkansas:

Escaping nontangentiality: Towards a controlled tangential amortized Julia-Carathéodory theory

Abstract: Let $f : D \to \Omega$ be a complex analytic function. The Julia quotient is given by the ratio between the distance of $f(z)$ to the boundary of Ω and the distance of z to the boundary of D. A classical Julia-Carathéodory type theorem states that if there is a sequence tending to τ in the boundary of D along which the Julia quotient is bounded, then the function f can be extended to τ such that f is nontangentially continuous and differentiable at τ and $f(\tau)$ is in the boundary of Ω. We develop an extended theory when D and Ω are taken to be the upper half plane which corresponds to amortized boundedness of the Julia quotient on sets of controlled tangential approach, so-called λ-Stolz regions, and higher order regularity, including but not limited to higher order differentiability, which we measure using γ-regularity. I will discuss the proof, along with some applications, including moment theory and the fractional Laplacian.

This is joint work with J.E. Pascoe and Ryan Tully-Doyle.

MARY ANGELICA TURSI, University of Illinois at Urbana-Champaign:

Separable Universal Banach Lattices

Abstract: The study of injective and projective universal objects spans across different classes of objects and has several applications. In this talk, we show that $C(\Delta, L_1)$ is injectively universal for the class of separable Banach lattices. The projectively universal separable Banach lattice is also constructed.

ELLEN WELD, Purdue University:

Connective Bieberbach Groups

Abstract: Bieberbach groups are discrete co-compact torsion free subgroups of the isometry group of $\mathbb{R}^n$. These groups describe the symmetries of crystals, are the fundamental groups of compact flat Riemannian manifolds, and have been extensively studied in math, physics, and chemistry. Connectivity is a geometric property of nuclear C^*-algebras that is equivalent to the unsuspension of E-theory of Connes and Higson. In this talk, we discuss when a Bieberbach group is connective and provide necessary and sufficient conditions for determining connectivity. This is joint work with Marius Dadarlat.