Fishing in Random Environments: Steady-State Conditions for More General Models

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Abstract

We have previously studied stochastic differential equation models of population growth and fishing in random environments based on a quite general density-dependent deterministic model

$$\frac{1}{N}\frac{dN}{dt} = rNg(N) - f(N), \qquad (*)$$

where $N = N(t)$ is the population size at time t, $r > 0$ is an intrinsic growth parameter, $g(N)$ is a smooth decreasing density-dependence natural growth function, and $f(N)$ is the fishing effort (assumed to be smooth and non-negative). We have considered the environmental random fluctuations to be well approximated by white noise $\sigma\varepsilon(t)$, where σ is an intensity parameter and $\varepsilon(t)$ is standard white noise. Two cases were considered: either the fluctuations affect the intrinsic growth parameter (replace r by $r + \sigma\varepsilon(t)$ in (*)) or they affect birth/death rates (add $\sigma\varepsilon(t)$ to the right side of (*)). We have obtained conditions on the fishing effort for the existence of a steady-state (either a stable equilibrium or a stationary density) and obtained their expressions.

In this paper we extend these results to more general models, allowing for non-smooth fishing policies (such as, for example, constant effort or constant quota fishing policies with a cut-off) and for more general ways of environmental fluctuations to affect population growth.