

THIS EXAM IS CLOSED TO BOOKS AND NOTES.
CALCULATORS ARE ALLOWED for the first 2 problems.
Use the back of the **previous** page if more space is needed!

MA303

EXAMINATION III (Practice)

Name _____ ID # _____ Section # _____

There are 10 Problems on this booklet. For All problems, mark the answers clearly.

Points awarded

1. (5 pts) _____

2. (5 pts) _____

3. (5 pts) _____

4. (5 pts) _____

5. (5 pts) _____

6. (5 pts) _____

7. (5 pts) _____

8. (5 pts) _____

9. (5 pts) _____

10. (5 pts) _____

Total Points: _____

1. (5 points) The approximate value of the solution at $t = 0.2$ of

$$y' = t^3 + y^2, \quad y(0) = 1$$

is evaluated using Euler method with $h = 0.1$. The approximate value is _____.

2. (5 points) Consider the equation

$$y' = t + y^2, \quad y(1) = 0.$$

The following Runge-Kutta method is used to compute an approximation to $y(3)$ with $h = 2$:

$$y_{n+1} = y_n + \frac{h}{6}(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4})$$

where

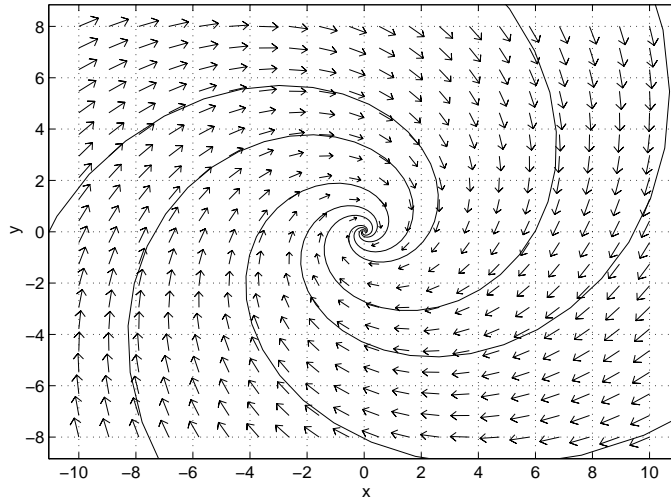
$$\begin{aligned} k_{n1} &= f(t_n, y_n), & k_{n2} &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n1}\right), \\ k_{n3} &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_{n2}\right), & k_{n4} &= f(t_n + h, y_n + hk_{n3}). \end{aligned}$$

The approximate value of $y(3)$ is _____.

3. (5 points) The phase portrait for a linear system of the form

$$\mathbf{x}' = A\mathbf{x},$$

where A is a 2×2 real matrix, is as follows.



Which of the following best describes the eigenvalues of A :

- a) two real negative eigenvalues
- b) two real eigenvalues of opposite signs
- c) two complex eigenvalues with positive real part
- d) two complex eigenvalues with negative real part
- e) one negative repeated eigenvalue

4. (5 points) Find all values of k for which the equilibrium point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ of the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & k \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

is a spiral (in) point.

- a) $k < 1$
 - b) $k > 1$
 - c) $k < 2$
 - d) $k > 2$
 - e) $k = -1$
5. (5 points) If $\mathbf{x}(t) = 4 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 2 \left[t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} \right]$, then $\mathbf{x}(2) =$

- a) $\mathbf{x}(2) = \begin{pmatrix} 10 \\ -7 \end{pmatrix}$
- b) $\mathbf{x}(2) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- c) $\mathbf{x}(2) = \begin{pmatrix} 12 \\ -10 \end{pmatrix}$
- d) $\mathbf{x}(2) = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$
- e) $\mathbf{x}(2) = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$

6. (5 points) The solution of the initial value problem $\mathbf{x}' = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is

a) $\mathbf{x}(t) = e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) $\mathbf{x}(t) = 2e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

c) $\mathbf{x}(t) = 3e^{3t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

d) $\mathbf{x}(t) = 2e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} - e^{2t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

e) $\mathbf{x}(t) = e^{2t} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

7. (5 points) Given that the matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ has complex eigenvalues $r_1 = 1 + i$ and $r_2 = 1 - i$, with corresponding complex eigenvectors $\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -i \end{pmatrix}$, find the general solution of $\mathbf{x}' = A\mathbf{x}$.

a) $\mathbf{x} = C_1 e^t \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$

b) $\mathbf{x} = C_1 e^t \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ -\cos(t) \end{pmatrix}$

c) $\mathbf{x} = C_1 e^t \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ -\cos(t) \end{pmatrix}$

d) $\mathbf{x} = C_1 e^t \begin{pmatrix} \cos(t) \\ -\sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$

e) $\mathbf{x} = C_1 e^t \begin{pmatrix} 2 \cos(t) \\ \sin(t) \end{pmatrix} + C_2 e^t \begin{pmatrix} \sin(t) \\ 2 \cos(t) \end{pmatrix}$

8. (5 points) If the general solution of the homogeneous system $\mathbf{x}' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x}$ is given by $\mathbf{x}_H(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then the general solution of the nonhomogeneous system $\mathbf{x}' = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} + e^{-t} \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ is

a) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

b) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

c) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

d) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

e) $\mathbf{x}(t) = C_1 e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

9. (5 points) Consider

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \mathbf{x}$$

Find an initial data $\mathbf{x}(0)$ such that when $t \rightarrow \infty$, the solution approaches $(0, 0)$.

a) $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

b) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

c) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d) $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

e) $\mathbf{x}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

10. (5 points) Find the general solution of the system $\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}$

a)

EXAMINATION III (Answer)

1. 1.2211

2. 172

3. D

4. B

5. C

6. B

7. A

8. E

9. D

10. $X(t) = c_1 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t} \left(t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix} \right)$