SOLUTIONS TO PRACTICE QUESTIONS FOR THE FINAL EXAM

1. $16x^2 - 4y^8 = 4\left(4x^2 - y^8\right) = 4\left((2x)^2 - \left(y^4\right)^2\right) = 4\left(2x - y^4\right)\left(2x + y^4\right)$

2. $\left(\dfrac{36a^{-4}b^{10}c^2}{a^2c^{-6}}\right)^{-1/2} = \left(36a^{-6}b^{10}c^8\right)^{-1/2} = (36)^{-1/2}\left(a^{-6}\right)^{-1/2}\left(b^{10}\right)^{-1/2}\left(c^8\right)^{-1/2} =$

$= \dfrac{1}{(36)^{1/2}}a^3b^{-5}c^{-4} = \dfrac{1}{\sqrt{36}}\dfrac{a^3}{b^5c^4} = \dfrac{a^3}{6b^5c^4}$

3. $\dfrac{3x}{3x+1} - \dfrac{x}{x-2} = \dfrac{3x}{(3x+1)}\dfrac{(x-2)}{(x-2)} - \dfrac{x}{(x-2)}\dfrac{(3x+1)}{(3x+1)} =$

$= \dfrac{3x(x-2) - x(3x+1)}{(3x+1)(x-2)} = \dfrac{3x^2 - 6x - 3x^2 - x}{(3x+1)(x-2)} = \dfrac{-7x}{(3x+1)(x-2)}$

4.

$(2x+1)^3(2)(3x-5)(3) + (3x-5)^2(3)(2x+1)^2(2)$

$= 6\left[(2x+1)^3(3x-5) + (3x-5)^2(2x+1)^2\right]$

$= 6(2x+1)^2\left[(2x+1)(3x-5) + (3x-5)^2\right]$

$= 6(2x+1)^2(3x-5)\left[(2x+1) + (3x-5)\right]$

$= 6(2x+1)^2(3x-5)(5x-4)$

$= 6(3x-5)(5x-4)(2x+1)^2$

5. $\dfrac{xy^{-1}}{(x+y)^{-1}} = \dfrac{\frac{x}{y}}{\frac{1}{(x+y)}} = \dfrac{x}{y}\dfrac{(x+y)}{1} = \dfrac{x(x+y)}{y}$

6.

$A = P(1+rt)$

$A = P + \Pr t$

$A - P = \Pr t$

$\dfrac{A-P}{\Pr} = \dfrac{\Pr t}{\Pr}$

$\dfrac{A-P}{\Pr} = t$

$t = \dfrac{A-P}{\Pr}$

7.

$\dfrac{4}{2p-3} + \dfrac{10}{4p^2-9} = \dfrac{1}{2p+3}$

$\dfrac{4}{2p-3} + \dfrac{10}{(2p-3)(2p+3)} = \dfrac{1}{2p+3}$

$(2p-3)(2p+3)\left[\dfrac{4}{2p-3} + \dfrac{10}{(2p-3)(2p+3)}\right] = \left[\dfrac{1}{2p+3}\right](2p-3)(2p+3)$

$(2p+3)\;4 + 10 = 1\;(2p-3)$

$4(2p+3) + 10 = (2p-3)$

$8p + 12 + 10 = 2p - 3$

$8p - 2p = -3 - 12 - 10$

$6p = -25$

$p = -\dfrac{25}{6}$

8. $\dfrac{\sqrt{x}+5}{\sqrt{x}-5} = \dfrac{\left(\sqrt{x}+5\right)\left(\sqrt{x}+5\right)}{\left(\sqrt{x}-5\right)\left(\sqrt{x}+5\right)} =$

$= \dfrac{x + 5\sqrt{x} + 5\sqrt{x} + 25}{x + 5\sqrt{x} - 5\sqrt{x} - 25} = \dfrac{x + 10\sqrt{x} + 25}{x - 25}$

9. t = # of hours the other person takes to complete the job.

fraction from 1st person + fraction from 2nd person=whole job

$\left(\dfrac{1}{6}\dfrac{job}{hour}\right)4hours + \left(\dfrac{1}{t}\dfrac{job}{hour}\right)4hours = \left(\dfrac{1}{4}\dfrac{job}{hour}\right)4hours$

$\left(\dfrac{2}{3}\right)job + \left(\dfrac{4}{t}\right)job = 1job$

$\dfrac{2}{3} + \dfrac{4}{t} = 1$

$3t\left(\dfrac{2}{3} + \dfrac{4}{t}\right) = 1\;3t$

$2t + 12 = 3t$

$12 = t$

$t = 12$

10.

$\begin{cases} y = x + 1 \\ y^2 - x^2 = 145 \end{cases}$

$(x+1)^2 - x^2 = 145$

$x^2 + 2x + 1 - x^2 = 145$

$2x + 1 = 145$

$2x = 144$

$x = 72$

11. let t = # hours truck has been traveling

$40t = 55(t-1)$

$40t = 55t - 55$

$55 = 15t$

	rate	time	distance
truck	40	t	40t
car	55	t - 1	55(t – 1)

$t = \dfrac{55}{15} = \dfrac{11}{3}$ hours, so distance is $40\left(\dfrac{11}{3}\right) = \dfrac{440}{3}$ miles

12.

let $x =$ # ml of the 50% solution

let $y =$ total # of ml

$\begin{cases} x + 40 = y \\ x(.50) + 40(.20) = y(.25) \end{cases}$

B

$x(.50) + 8 = (x + 40)(.25)$

$.50x + 8 = .25x + 10$

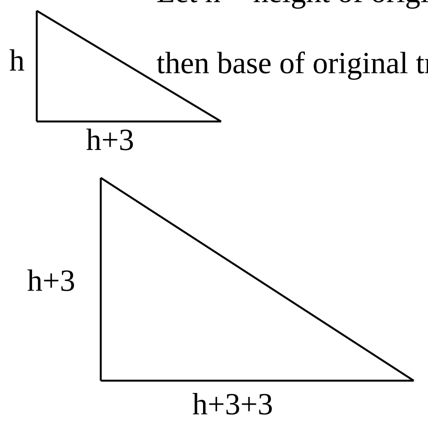
$.25x = 2$

$x = 8$ ml

13.

Let h = height of original triangle

then base of original triangle = h+3



New:

Area of new triangle $= 14\ in^2$

$\frac{1}{2}(h+3)(h+6) = 14$

$(h+3)(h+6) = 28$

$h^2 + 3h + 6h + 18 = 28$

$h^2 + 9h - 10 = 0$

$(h+10)(h-1) = 0$

$h = -10, \quad h = 1$

Original height $= 1\ in.$

Original base $= 1 + 3 = 4\ in.$

A

14.

let t = number of years after 1980 and let V =value
t is the independent variable and V is the dependent variable

points on line $(1,54)$ and $(3,62)$

slope $m = \frac{62-54}{3-1} = \frac{8}{2} = 4$

$V - V_1 = m(t - t_1)$

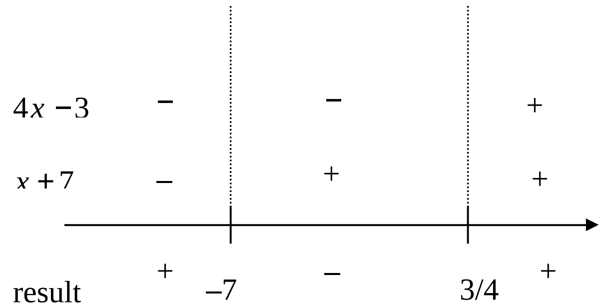
$V - 54 = 4(t-1)$

$V - 54 = 4t - 4$

$V = 4t + 50$

A

15. $(4x-3)(x+7) \le 0$



Answer: $\left[-7, \frac{3}{4}\right]$

16.

$|6 - 2x| \le 3$

$-3 \le 6 - 2x \le 3$

$-9 \le -2x \le -3$

$\frac{9}{2} \ge x \ge \frac{3}{2}$

$\frac{3}{2} \le x \le \frac{9}{2}$

C

17.

$A\ (1,-2)$, Midpoint $M\ (2,3)$, $B\ (x,y)$

Midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of $\overline{AB}$ $\left(\frac{1+x}{2}, \frac{-2+y}{2}\right)$ $(2,3)$

$\frac{1+x}{2} = 2, \qquad \frac{-2+y}{2} = 3$

$1 + x = 4, \qquad -2 + y = 6$

$x = 3, \qquad y = 8$

so $B\ (3,8)$

C

18.

slope of line $m = -\frac{1}{3}$

slope of line perpendicular $m = 3$

D

19.

$2x - 3y = 7$

$-3y = -2x + 7$

$y = \frac{2}{3}x - \frac{7}{3}$

slope $m = \frac{2}{3}$

slope of parallel line $m = \frac{2}{3}$

point is $(2,-1)$; $m = \frac{2}{3}$

$y = mx + b$

$-1 = \left(\frac{2}{3}\right)(2) + b$ **C**

$-1 = \frac{4}{3} + b$

$b = -\frac{7}{3}$ so $y = \frac{2}{3}x - \frac{7}{3}$

20.

Center $(0,2)$ $(x-h)^2 + (y-k)^2 = r^2$

radius = 2 $(x-0)^2 + (y-2)^2 = 2^2$

$x^2 + (y-2)^2 = 4$ **B**

$x^2 + y^2 - 4y + 4 = 4$

$x^2 + y^2 - 4y = 0$

21.

$f(x) = 1 - \sqrt{x}$, $g(x) = \frac{1}{x}$

D

$(g \circ f)(x) = g[f(x)] = g(1 - \sqrt{x}) = \frac{1}{1 - \sqrt{x}}$

22.

$f(x) = \frac{x}{x^2 + 1}$

D

$\frac{1}{f(3)} = \frac{1}{\frac{3}{(3)^2 + 1}} = \frac{1}{\frac{3}{10}} = \frac{10}{3}$

23.

$y = \frac{1}{3x - 2}$

$x = \frac{1}{3y - 2}$

$x(3y - 2) = 1$

$3xy - 2x = 1$

$3xy = 1 + 2x$

$y = \frac{1 + 2x}{3x} = f^{-1}(x)$

24.

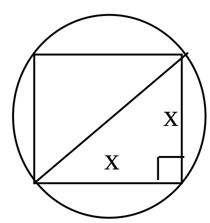
$f(x) = x^2 - 2x + 4$

$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$ **A**

$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 4 - x^2 + 2x - 4}{h} = \frac{2xh + h^2 - 2h}{h}$

$= \frac{h(2x + h - 2)}{h} = 2x + h - 2$

25.



Let A = area of circle

Area of circle $A(r) = \pi r^2$

Diameter (d) of circle $x^2 + x^2 = d^2$

$2x^2 = d^2$ **A**

$d = \pm\sqrt{2x^2}$

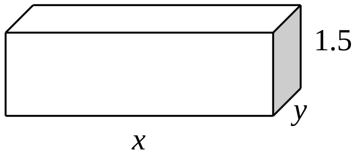
$d = x\sqrt{2}$

Radius (r) of circle $r = \frac{x\sqrt{2}}{2}$

So, $A(x) = \pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \pi\left(\frac{x^2(2)}{4}\right)$

$= \frac{\pi x^2}{2}$ or $\frac{\pi}{2}x^2$

26.



Volume $= 6\,ft.^3$

$xy(1.5) = 6$

$y = \dfrac{6}{1.5x}$ **B**

$y = \dfrac{4}{x}$

27.

$x^2 - 4x - 2y - 4 = 0$

$2y = x^2 - 4x - 4$

$2y = (x^2 - 4x + 4) - 4 - 4$

$2y = (x-2)^2 - 8$

$y = \frac{1}{2}(x-2)^2 - 4$

$y = a(x-h)^2 + k$

$Vertex(h,k) = (2,-4)$

28.

Vertex $V(0,2)$ $y = a(x-h)^2 + k$

point on parabola (1,0) $y = a(x-0)^2 + 2$

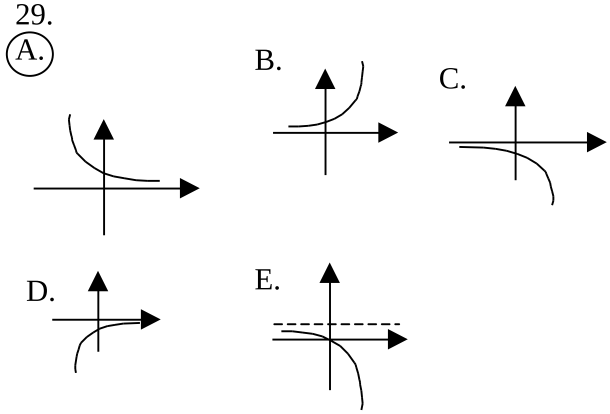
$y = ax^2 + 2$ **B**

$0 = a(1)^2 + 2$

$a = -2$

$y = -2x^2 + 2$

29.



30.

$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b(y^3y^2) - \log_b y^4$

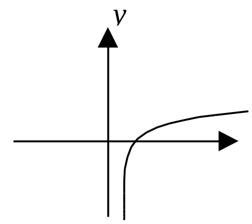
$= \log_b y^5 - \log_b y^4 = \log_b\left(\dfrac{y^5}{y^4}\right) = \log_b y$ **B**

31.

$f(x) = \log_a x$ if $a > 1$

example: if $a = 2$, then $f(x) = \log_2 x$, **D**

Graph of $y = \log_2 x$ $2^y = x$



f is increasing, f does not have a as an x-intercept (the x-int. is (1,0)), f does not have a y-intercept, the domain of f is $(0,\infty)$.

32.

$\log\left(\dfrac{432}{\left(\sqrt{.095}\right)\left(\sqrt[3]{72.1}\right)}\right) = \log\left(\dfrac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}}\right)$

$= \log 432 - \left(\log\left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}\right]\right)$ **B**

$= \log 432 - \left(\log(.095)^{\frac{1}{2}} + \log(72.1)^{\frac{1}{3}}\right)$

$= \log 432 - \dfrac{1}{2}\log .095 - \dfrac{1}{3}\log 72.1$

33.

$\log_x 2 = 5$

$x^5 = 2$

$\left(x^5\right)^{\frac{1}{5}} = (2)^{\frac{1}{5}}$ **D**

$x = \sqrt[5]{2}$

$x \approx 1.1487$

34.

$\dfrac{\log_5\left(\frac{1}{8}\right)}{\log_5(2)} = \log_2\left(\frac{1}{8}\right) = \log_2\left(2^{-3}\right) = -3$

35.

$3^{x-5} = 4$

$\log 3^{x-5} = \log 4$

$(x-5)\log 3 = \log 4$

$x - 5 = \dfrac{\log 4}{\log 3}$

$x = \dfrac{\log 4}{\log 3} + 5$

C

36.

$\log_3 \sqrt{2x+3} = 2$

$3^2 = \sqrt{2x+3}$

$\sqrt{2x+3} = 9$

$\left(\sqrt{2x+3}\right)^2 = (9)^2$

$2x + 3 = 81$

$2x = 78$

$x = 39$

Check: $\sqrt{2(39)+3} = 9$

$9 = 9$

Check: $\log_3 \sqrt{2(39)+3} = 2$

$3^2 = \sqrt{81}$

C

37.

$\log_3 m = 8$

$\log_3 n = 10$

$\log_3 p = 6$

$\log_3\left(\dfrac{\sqrt{mn}}{p^3}\right) = \log_3 (mn)^{\frac{1}{2}} - \log_3 p^3$

$= \log_3\left(m^{\frac{1}{2}} n^{\frac{1}{2}}\right) - \log_3 p^3$

$= \log_3 m^{\frac{1}{2}} + \log_3 n^{\frac{1}{2}} - \log_3 p^3$

$= \dfrac{1}{2}\log_3 m + \dfrac{1}{2}\log_3 n - 3\log_3 p$

$= \dfrac{1}{2}(8) + \dfrac{1}{2}(10) - 3(6)$

$= 4 + 5 - 18 = -9$

A

38. Half-life means when half of te initial amount still remains, $\frac{1}{2}q_o$.

$\frac{1}{2}q_o = q_o e^{-0.0063t}$

$\frac{1}{2} = e^{-0.0063t}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-0.0063t}\right)$

$\ln\left(\frac{1}{2}\right) = -0.0063t$

$\dfrac{\ln(0.5)}{-0.0063} = t \approx 110.0$ days

39.

$y = 2 + 2^x$

When $x = 0$, $y = 2 + 2^0$

$y = 2 + 1 = 3$

D

40.

$\begin{cases} x + 4y = 3 \\ 2x - 6y = 8 \end{cases}$ $\quad x = 3 - 4y$

$2(3 - 4y) - 6y = 8$

$6 - 8y - 6y = 8$

$y = \frac{2}{-14} = -\frac{1}{7} \qquad x = 3 - 4\left(-\frac{1}{7}\right) = \frac{25}{7}$

$\left(\frac{25}{7}, -\frac{1}{7}\right)$

41.

$\begin{cases} x^2 + y^2 = 16 \\ 2y - x = 4 \end{cases}$ $\quad x = 2y - 4$

$(2y-4)^2 + y^2 = 16$

$4y^2 - 16y + 16 + y^2 = 16$

$5y^2 - 16y = 0$

$y(5y - 16) = 0$

$y = 0 \qquad x = 2(0) - 4 = -4$

$y = \frac{16}{5} \qquad x = 2\left(\frac{16}{5}\right) - 4 = \frac{12}{5}$

$(-4, 0)$ & $\left(\frac{12}{5}, \frac{16}{5}\right)$

42.

$$\begin{array}{r} x^2 + 6x + 34 \\ x^2 - 6x + 0 \,\big)\, x^4 + 0x^3 - 2x^2 + 0x - 3 \\ +\left(^{-}x^4 + 6x^3 + {}^{-}0x^2\right) \\ \hline 6x^3 - 2x^2 + 0x \\ +\left(^{-}6x^3 + 36x^2 + {}^{-}0x\right) \\ \hline 34x^2 + 0x - 3 \\ +\left(^{-}34x^2 + 204x + {}^{-}0\right) \\ \hline 204x - 3 \end{array}$$

$q(x) = x^2 + 6x + 34$

$r(x) = 204x - 3$

C

43. If the denominator of the function is equal to zero the function will be undefined.

$$f(x) = \frac{(x+3)(x-3)}{x(x+2)}$$

when $x = 0$ or $x = -2$

44.

$$y = x^2(x-1)(x+1)^2$$

$$\text{x-intercepts:} x^2(x-1)(x+1)^2 = 0$$

$$x = 0,\ x = 1,\ x = -1$$

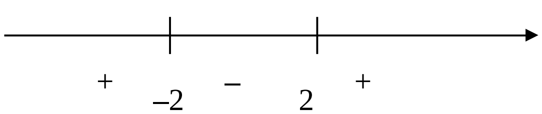
A

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
x^2	+	+	+	+
$x-1$	−	−	−	+
$(x+1)^2$	+	+	+	+
Result	−	−	−	+
	below x-axi	below x-axi	below x-axi	above x-axi

45.

$x = 2$ x-intercept

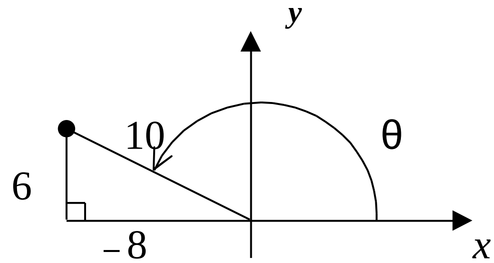
$x = -2$ vertical asymptote



E is the closest answer. The scale is a bit off on the *x*-axis.

46. Shifted left 1 unit, then reflected about x-axis, then shifted down 2 units -- Answer: C

47.



$$\sin\theta = \frac{6}{10} = \frac{y}{r}$$

$$x^2 + y^2 = r^2$$

$$x^2 + 6^2 = 10^2$$

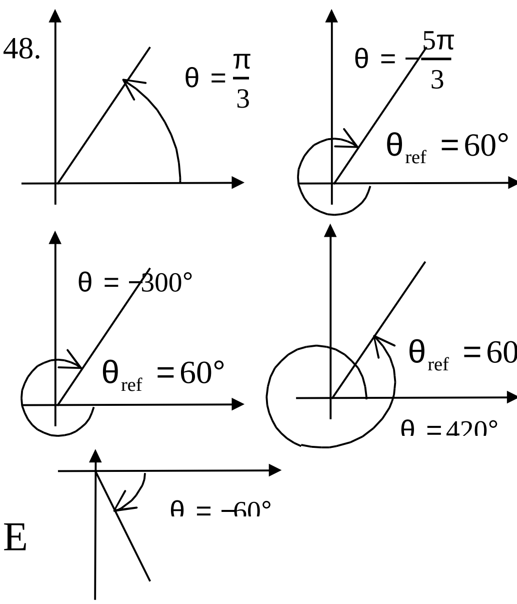
$$x^2 = 64$$

$$x = \pm 8$$

$$x = -8$$

$$\cos\theta = \frac{x}{r} = \frac{-8}{10} = -0.8$$

48.



E

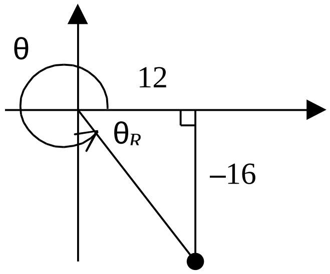
49.

$$135° \ \frac{\pi \text{ radians}}{180°} = \frac{3\pi}{4}$$

50.

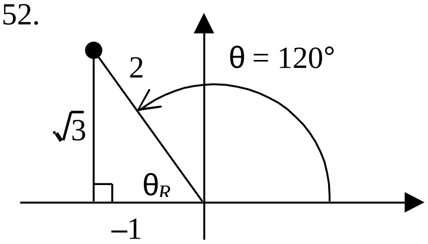
$$\sec 126° = \frac{1}{\cos 126°} \approx \frac{1}{-0.587785} \approx -1.7013$$

51.



$\tan\theta = \frac{y}{x} = \frac{-16}{12} = -\frac{4}{3}$

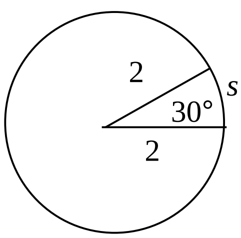
52.



$\theta_R = 60°$

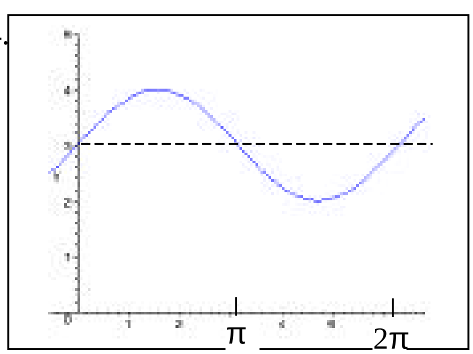
$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

53.



$s = r\theta = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} \approx 1.047$

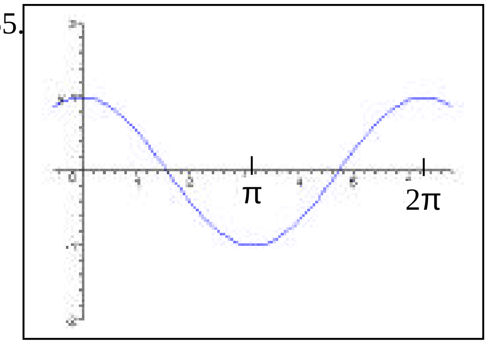
54.



The graph is the $y = \sin x$ shifted up three units.

I(yes), II(no), III(yes), IV(yes), **B**

55.



D = all real number = $(-\infty, \infty)$

R = all possible outputs/y-values = $[-1, 1]$

56.

$$\frac{\tan^2 x}{1+\sec x} = \frac{\sec^2 x - 1}{\sec x + 1} = \frac{(\sec x + 1)(\sec x - 1)}{(\sec x + 1)}$$

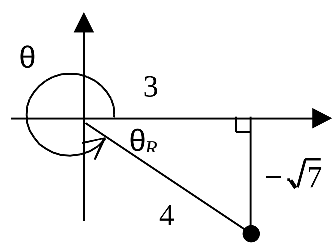
$= \sec x - 1$, remember $\tan^2 x = (\tan x)^2$

57.

$$\frac{\tan x \ \cos x \ \csc x}{\cot x \ \sec x \ \sin x} = \frac{\tan x \ \tan x \ \cos x \ \cos x}{\sin x \ \sin x}$$

$$= \frac{\tan^2 x \ \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = 1$$

58.



$x^2 + y^2 = r^2$

$y^2 = 4^2 - 3^2$

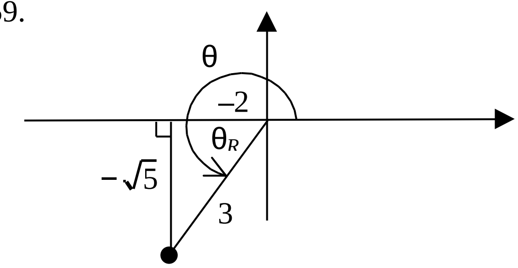
$y = \pm\sqrt{7}$

$y = -\sqrt{7}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-\sqrt{7}}{4}\right)\left(\frac{3}{4}\right) = -\frac{3\sqrt{7}}{8}$

59.

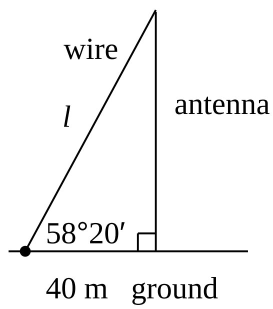


Given: $\tan\theta = \frac{\sqrt{5}}{2} = \frac{-\sqrt{5}}{-2} = \frac{y}{x}$

$\frac{\theta}{2}$ is in QII, $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\left(\frac{-2}{3}\right)}{2}} = -\sqrt{\frac{1-\frac{2}{3}}{2}}$

60.



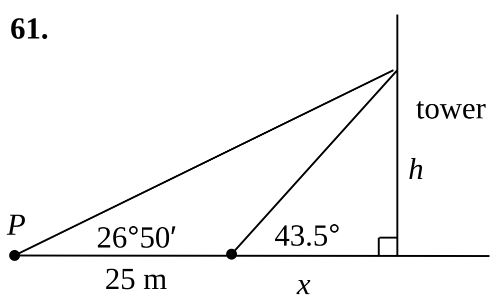
$\cos 58.\overline{3}^\circ = \frac{40}{l}$

$l \cos 58.\overline{3}^\circ = 40$

$l = \frac{40}{\cos 58.\overline{3}^\circ}$

$l \approx 76.2$ m

61.



$\tan 43.5^\circ = \frac{h}{x}, \quad \tan 26.8\overline{3}^\circ = \frac{h}{x+25}$

$h = x \tan 43.5^\circ$

$\tan 26.8\overline{3}^\circ = \frac{x \tan 43.5^\circ}{x+25}$

$\tan 26.8\overline{3}^\circ (x+25) = x \tan 43.5^\circ$

$x \tan 26.8\overline{3}^\circ + 25\tan 26.8\overline{3}^\circ = x \tan 43.5^\circ$

$25\tan 26.8\overline{3}^\circ = x \tan 43.5^\circ - x\tan 26.8\overline{3}^\circ$

$x = \frac{25\tan 26.8\overline{3}^\circ}{\tan 43.5^\circ - \tan 26.8\overline{3}^\circ} \approx 28.541487$

$h = x \tan 43.5^\circ \approx 27.1$ meters

62. Examine r when $\theta = 0$ and as $\theta \to 90^\circ$

A. $r = 1$ when $\theta = 0$ and as $\theta \to 90^\circ$ $r \to 2$, looks right as the angle changes from 0° to 90°.

B. $r = 2$ when $\theta = 0$ and as $\theta \to 90^\circ$ $r \to 1$, looks incorrect as the angle changes from 0° to 90°. The radial distance should be getting bigger.

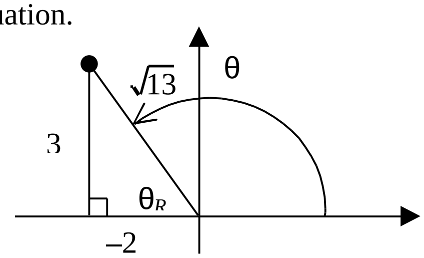
C. $r = 1$ when $\theta = 0$ and as $\theta \to 90^\circ$ $r \to 0$, looks incorrect as the angle changes from 0° to 90°. The radial distance should be getting bigger.

D. $r = 2$ when $\theta = 0$ and as $\theta \to 90^\circ$ $r \to 0$, looks incorrect as the angle changes from 0° to 90°. The radial distance should be getting bigger.

E. $r = 0$ when $\theta = 0$ and as $\theta \to 90^\circ$ $r \to 2$, looks incorrect as the angle changes from 0° to 90°. When the angle is zero the radial distance should greater than zero.

Plugging in further angles would yield more points that will confirm that **A** is the correct polar equation.

63.



$r^2 = (-2^2) + 3^2$

$r = \sqrt{13}$

$\tan\theta = \frac{y}{x} = \frac{3}{-2} \qquad \tan^{-1}\left(-\frac{3}{2}\right) \approx -56.301^\circ$

$\theta_R = +56.301^\circ$

$\theta = 180^\circ - \theta_R \approx 123.7^\circ$

$\left(\sqrt{13}, 123.7^\circ\right)$

64.

$x^2 - 2x + y^2 = 0$

$x^2 + y^2 - 2x = 0$

$r^2 - 2r\cos\theta = 0$

$r(r - 2\cos\theta) = 0$

$r = 0$, which is not an equation of the given circle

or $\underline{r = 2\cos\theta}$