MA 265 Practice Test 1 (Dr. Park)

Name . . . .

ID number . . . .

Test 1: Feb 20, 2020

INSTRUCTIONS in the Test

1. **Do not open this exam booklet until told to do so**.

2. **Show all your work - if you need more space, continue on the back of the page for that problem.**

3. **Show your final answer by enclosing it in a box or circle**.

4. **If in multiple-choice problems your work is not directly related to the correct answers, no partial credit will be given.**

Problem 1. Let A and B be $n \times n$ matrices. Which of the following statements are always TRUE?

(a) $(-A)^T = -A^T$ (b) $(A - B)^T = A^T - B^T$ (c) $(A^T B)^T = AB^T$

(d) $(AB^{-1})^{-1} = A^{-1}B$ (e) $AB = BA$

Answer: $(a), (b)$

Problem 2. Let A, B and C be arbitrary $n \times n$ matrices. What is the transpose of the matrix?

$$(A^T + 2020B)C^{-1}$$

Answer: $(C^{-1})^T(A + 2020B^T)$

Problem 3. Let A be a nonsingular matrix with its inverse:

$$A^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 5 \\ -1 & 5 & 0 \end{bmatrix}$$

Which of the following statements is FALSE?

(a) For arbitrary 3×3 matrices B and C if $BA = BC$, then $A = C$.
(b) A^T is invertible.
(c) For arbitrary 3×3 matrices B and C if $AB = AC$, then $B = C$.
(d) $AA^{-1} = A^{-1}A$.

Answer: (a)

Problem 4. Let

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 4 \\ 2 & a & 10 \end{bmatrix}.$$

Find the value(s) of a such that the columns of A are linearly dependent.

Answer: $a = 4$

Problem 5. For what values of h and k does the system $A\mathbf{x} = \mathbf{b}$ have infinitely many solutions?

$$A = \begin{bmatrix} 1 & 1 & 4 \\ -3 & -3 & h \\ 1 & 8 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -2 \\ k \\ 0 \end{bmatrix}.$$

Answer: $h = -12,\ k = 6$

Problem 6. Let A, B and C be invertible $n \times n$ matrices. If $A^{-1}B^{-1} = C^{-1}$, then what is A?

(a) $A = CB^{-1}$ (b) $A = C^{-1}B^{-1}$ (c) $A = BC^{-1}$ (d) $A = B^{-1}C$
(e) $A = BC$

Answer: (d)

Problem 7. Which of the following statements are true?

(i) A linear system of four equations in three unkowns is always inconsistent.
(ii) A linear system with fewer equations than unkowns must have infinitely many solutions.
(iii) If for any $\mathbf{b} \in \mathbb{R}^n$ the system $AX = \mathbf{b}$ has a unique solution, then A must be a square matrix.
(iv) For a square matrix A if $AX = \mathbf{0}$ has a nonzero solution, then the homogeneous linear system has infinitely many solutions.

(a) all of them (b) (i) and (ii) (c) (ii) and (iii) (d) (iii) and (iv)
(e) (iv) only.

Answer: (d)

Problem 8. Let

$$A^T = \begin{bmatrix} 2 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{bmatrix}.$$

Find the $(3,1)$ entry of AA^T.

Answer: 108

Problem 9. Find the solution (x_1, x_2, x_3) of the linear system of equations

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ 2x_1 + 4x_2 + 7x_3 &= 2 \\ 3x_1 + 10x_2 + 5x_3 &= 7 \end{aligned}$$

Answer: $x_1 = -59/9$; $x_2 = 20/9$; $x_3 = 8/9$

Problem 10. Consider the linear system $AX = \mathbf{b}$ where A is a 3×2 matrix and $\mathbf{b}$ is in $\mathbb{R}^3$. Which of the following statements is TRUE for every matrix A?

(a) The equation $AX = \mathbf{b}$ is inconsistent for every $\mathbf{b}$ in $\mathbb{R}^3$.
(b) Whenever the equation $AX = \mathbf{b}$ is consistent, it has exactly one solution$X$.
(c) Whenever the equation $AX = b$ is consistent, it has infinitely many solutions X.
(d) The equation $AX = \mathbf{b}$ is inconsistent for at least one $\mathbf{b}$ in $\mathbb{R}^3$.
(e) If the columns of A are a scalar multiple of one another, then the equation $AX = \mathbf{b}$ has exactly one solution.

Answer: (d)

Problem 11. Let T be the linear transformation whose standard matrix is

$$A = \begin{bmatrix} 1 & 4 & 8 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Which of the following statement is TRUE?

(a) T maps $\mathbb{R}^3$ onto $\mathbb{R}^4$
(b) T maps $\mathbb{R}^4$ onto $\mathbb{R}^3$
(c) T is one-to-one
(d) The columns of the matrix A are linearly independent.

Answer: (b)

Problem 12. Find the values of α for which A is singular:

$$A = \begin{bmatrix} 2 & 1 & 3\alpha & 4 \\ 0 & \alpha - 1 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & \alpha & 4 \end{bmatrix}.$$

Answer: $\alpha = 1$ or 8

Problem 13. Let A be an $n \times n$ matrix, and let $\mathbf{R}^n$ be the vector space of all the vectors in the n-dimensional space. Prove that the following subsets of $\mathbf{R}^n$ are its subspaces.

(a) $\mathbf{U} = \{\mathbf{X} \in \mathbf{R}^n : A\mathbf{X} = \mathbf{0}\}$ is the set of all the solutions of the homogeneous system $A\mathbf{X} = \mathbf{0}$.

(b) $\mathbf{W} = \{\mathbf{X} \in \mathbf{R}^n : \mathbf{X}$ is a linear combination of the columns of $A\}$ is the set of all the vectors that can be written as a linear combination of the columns of A.

Answer: They were proved in class

Problem 14. Which of the following set of vectors span $\mathbb{R}^2$ and are linearly independent?

$(a) \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ $(b) \left\{ \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 10 \end{bmatrix} \right\}$

$(c) \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ $(d) \left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right\}$

Answer: (c)

Problem 15. Let $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be a linear transformation such that

$$L\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} \qquad \text{and} \qquad L\left(\begin{bmatrix} 3 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Find $L\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)$

Answer: $L([-1, 2]^T) = 2L([1, 2]^T) - L([3, 2]^T) = [2, 2, 7]^T$

Problem 16. Let A be an $m \times n$ matrix. Which of the following statements is ALWAYS TRUE?

(a) The nullity of A is the same as the nullity of A^T.
(b) The rank of A is the same as the rank of A^T.
(c) The column space of A is the same as the null space of A^T.
(d) The columns of A form a basis of the column space of A.
(e) The columns of A^T form a basis of the null space of A.

Answer: (b)