MA 265 Practice Test 2 (Dr. Park)

Name . . . .

ID number . . . .

Test 2: 8:00pm – 9:00pm April 2, 2020 (Eastern time in USA)

INSTRUCTIONS in the Test

1. **This test is open note and open book**. Do not use any online resources.

2. **You have to solve problems by yourself**.

3. **Show your final answer by enclosing it in a box or circle**.

4. **If in multiple-choice problems your work is not directly related to the correct answers, no partial credit will be given.**

5. **This test will be open at 8:00pm, April 2 (Thu) in the Eastern time (USA)**.

Problem 1. Let A be a nonsingular matrix with its inverse

$$A^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

Which of the following statements is FALSE?

(A) A is not diagonalizable.
(B) A^T is invertible.
(C) For arbitrary 2×2 matrices B and C if $AB = AC$, then $B = C$.
(D) $AA^{-1} = A^{-1}A$.
(E) A is symmetric.

Problem 2. Let

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 1 & 0 & 2 \\ -3 & 1 & -3 \end{bmatrix}$$

and let its inverse $A^{-1} = [b_{ij}]$. Find the trace of the matrix, A^{-1}. In other words, compute the sum $b_{11} + b_{22} + b_{33}$.

(A) -1 (B) 0 (C) 1/2 (D) 1 (E) 2

Problem 3. Assume that the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & a & 1 \end{bmatrix}$$

is invertible. What is the $(2, 1)$-entry of the inverse of A?

(A) $-2/(4a-1)$ (B) $(2-3a)/(4a-1)$ (C) $2/(4a-1)$

(D) $(-2+3a)/(4a-1)$ (E) $(-2+3a)/(4a+1)$

Problem 4. Given constants a and b, consider the linear system

$$\begin{aligned} 9x + 2y &= a \\ 4x + y &= b. \end{aligned}$$

Find the solution of the linear system by using Cramer's rule.

Problem 5. Given that

$$\det \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} = 4,$$

what is the determinant of

$$\det \begin{bmatrix} a_4 & a_2 & a_3 & a_1 \\ b_4 & b_2 & b_3 & b_1 \\ 2c_4 + 3a_4 & 2c_2 + 3a_2 & 2c_3 + 3a_3 & 2c_1 + 3a_1 \\ d_4 & d_2 & d_3 & d_1 \end{bmatrix}?$$

Problem 6. If A is a 3×3 matrix with $\det A = 5$ and $B = 2A$, then what is $\det\left(A^T B^{-1}\right)$?

Problem 7. Let $\mathcal{P}_3$ be the vector space of all polynomials of degree ≤ 3. Which of the following set(s) is(are) subspace(s) of $\mathcal{P}_3$?

(A) $\{1 + t^2\}$

(B) $\{at + bt^2 + (a+b)t^3 \in \mathcal{P}_3 : \ a \text{ and } b \text{ are real numbers}\}$

(C) $\{a + bt + abt^2 \in \mathcal{P}_3 : \ a \text{ and } b \text{ are real numbers}\}$

(D) $\{p(t) \in \mathcal{P}_3 : \ p(2) = 0\}$

Problem 8. Let

$$A = \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 3 & 2 & 3 & 1 & 3 \\ 3 & 1 & 2 & 2 & 2 \end{bmatrix}$$

Which of the following is a basis of the null space of A?

(A) $\begin{bmatrix} -4 \\ -8 \\ 9 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} 4 \\ 8 \\ -11 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -8 \\ -16 \\ 19 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \\ 0 \\ 1 \\ 9 \end{bmatrix}$

Problem 9. Compute the value of the following determinant:

$$\begin{bmatrix} 4 & -9 & 2 & 3 \\ 0 & 3 & 0 & -4 \\ -5 & 0 & 0 & 3 \\ 0 & 5 & 0 & -7 \end{bmatrix}$$

(A) 10 (B) -10 (C) 410 (D) -410 (E) 90

Problem 10. Suppose that $A = PDP^{-1}$, where P is a 3×3 invertible matrix and

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

Let $B = 2I + 3A + A^2$, which of the following is true?

(A) B is not diagonalizable

(B) B is diagonalizable and $B = PCP^{-1}$, where $C = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(C) B is diagonalizable and $B = PCP^{-1}$, where $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

(D) B is diagonalizable and $B = PCP^{-1}$ for some C, but there is not enough information to determine C.

(E) There is not enough information to determine whether B is diagonalizable.

Problem 11. Which of the following statements are true?

(i) If λ is an eigenvalue for A, then $-\lambda$ is an eigenvalue for $-A$.

(ii) If zero is an eigenvalue of A, then A is not invertible.

(iii) If an $n \times n$ matrix A is diagonalizable, then A has n distinct eigenvalues.

(iv) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. Then A is both invertible and diagonalizable.

(A) (i) and (ii) only (B) (i) and (iii) only (C) (i), (ii) and (iii) only

(D) (i), (ii) and (iv) only (E) (i), (ii), (iii) and (iv)

Problem 13. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(X) = AX$, where $A = \begin{bmatrix} 2 & -6 \\ -1 & 3 \end{bmatrix}$. Which of the following is a basis $\mathcal{B}$ for $\mathbb{R}^2$ with the property that the $\mathcal{B}$-matrix for T is a diagonal matrix?

(A) $\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$ (B) $\left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$

(C) $\left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}$ (D) $\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

Problem 14. Let A be an $n \times n$ matrix. Which of the following statements is(are) NOT equivalent to that A is invertible?

(i) Columns of A are linearly independent.

(ii) A is diagonalizable.

(iii) The dimension of the null space of A is 0.

(iv) $\det A = 0$

(A) (ii) only (B) (iv) only (C) (ii), (iii) only (D) (ii), (iv) only

(E) (iii), (iv) only

Problem 15. Which of the following matrices are diagonalizable?

(i) $\begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 6 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & 0 & 3 \\ 0 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

(A) (i) and (ii) only (B) (iii) and (iv) only (C) (i) and (iii) only

(D) (i), (iii) and (iv) only (E) (i), (ii), (iii) and (iv)