Eddie Price Fast Method for 3×3 Determinants Spring 2017

The method of cofactor expansion is the proper way to find a determinant for square matrices that are larger than 2×2 . For 3×3 matrices, specifically, we can use the following technique. The follow technique does *not* work for square matrices of other sizes.

Take the matrix you're starting with, copy the first two *columns* of your matrix, and place them to the right of your starting matrix. This new array with 5 columns will have 3 diagonals of length 3 (a diagonal moves down and to the right) and 3 anti-diagonals of length 3 (an anti-diagonal moves up and to the right). For each diagonal and anti-diagonal, multiply together all three of the entries on it. Then add all of the products obtained from the diagonals and subtract all of the products obtained from the anti-diagonals. The number you get is the determinant.

As an example, consider the matrix A below:

$$A = \begin{bmatrix} 1 & -1 & 2\\ 2 & 1 & 4\\ -1 & -1 & -2 \end{bmatrix}$$

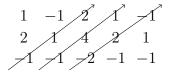
Now, copy the first two columns and place them to the right of the matrix:

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ -1 & -1 \end{bmatrix}$$

Look at the three diagonals of length 3:

Multiply along the diagonals to get (1)(1)(-2) = -2, (-1)(4)(-1) = 4, and (2)(2)(-1) = -4.

Now, look at the three anti-diagonals of length 3:



Multiply along the anti-diagonals to get (-1)(1)(2) = -2, (-1)(4)(1) = -4, and (-2)(2)(-1) = 4.

Now, add all of the products from diagonals and subtract all of the products of anti-diagonals to get

$$\det A = (-2) + (4) + (-4) - (-2) - (-4) - (4) = 0$$