Eddie Price Gaussian/Gauss-Jordan Elimination Spring 2017

Throughout the semester, we've encountered many topics without any sort of algorithmic way to solve them. Take integrals, for example. Each integral must be analyzed carefully to find a technique that will allow us to do the computation. The techniques in linear algebra are very different from that, however. They are very systematic. Gaussian and Gauss-Jordan Elimination are very algorithmic processes. Computers are very good at doing the techniques of linear algebra.

The first thing you always do in Gaussian and Gauss-Jordan Elimination is get the (1,1)entry to be 1. Often, this is done by switching rows if the first entry in some row is a 1, or you can multiply row one by the reciprocal of the first entry if it is not 0. After you get this first entry to be a 1, you want to use this 1 to get all of the entries below it to be 0. Suppose you want to make the first entry in row 2 be a 0, and the first entry in row 2 is c. You can take $-cR_1 + R_2$ to replace row 2. You repeat this process until column one starts with a 1 and everything below it is a 0.

Next, you want to make the (2,2)-entry into a 1. Again, you can do this by switching rows if one of the rows below row 2 has a 1 in column 2 or by multiplying row 2 by the reciprocal of the 2nd entry of row 2. Now that you have a 1 in row 2 column 2, you want to use this 1 to get all entries below it to be 0. Again, you do the same process as before, but instead with row 2 instead of row 1. If row 3, for instance starts with a 0 in column 1 and has a c in column 2, you can take $-cR_2 + R_3$ replacing row 3.

You repeat this general procedure until you get to row-echelon form.

If you are doing Gauss-Jordan elimination, you work your way back up the matrix and basically do the Gaussian Elimination process in reverse. You have a 1 in row 3 column 3 (and 0's everywhere else in row 3). So you use the 1 in row 3 to turn the entries above row 3 column 3 into 0's. Again, if the 3rd entry in row 2, for example is c, you can take $-cR_3 + R_2$ replacing row 2.

You repeat this process until you have reduced row-echelon form.

If you want to see this process in action, take a look at the Quiz 17 solutions posted on my website. You can see this basic process played out there. See how I first try to get a 1 in the appropriate place, and then I use that 1 to get entries below it to be 0. Then I try to get the (2,2)-entry to be 1, and I use that 1 to get the entries below it to be 0. etc.