MA 16020
Lesson 17
Geometric Series (Part 2)

For applications, write out a series, determine if you need the "first term," and find the sum.

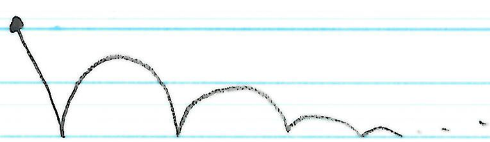
Ex 1. A patient is given an injection of 55 mg every 24 hours. After t days, the fraction of the drug remaining is $f(t) = 2^{-t/3}$. If the injections continue indefinitely, how much will be in the patient's body in the long run just prior to an injection? (Round 1 decimal)

$$\underbrace{55\cdot 2^{-1/3}}_{\text{1 day old}} + \underbrace{55\cdot 2^{-2/3}}_{\text{2 days old}} + \underbrace{55\cdot 2^{-3/3}}_{\text{3 days old}} + \cdots$$

$a = 55\cdot 2^{-1/3}$, $r = 2^{-1/3}$ $\quad \dfrac{a}{1-r} = \dfrac{55\cdot 2^{-1/3}}{1-2^{-1/3}} \approx \boxed{211.6\text{ mg}}$

(Similar to Mars Colony Problem)

Ex 2. Every time a ball bounces, it bounces to a height of rh, where h was the height it fell from. Find the total distance the ball travels if $r = 0.7$ and it is dropped from a height of 12 m.



$12 + 0.7\cdot 12 + 0.7\cdot 12 + 0.7^2\cdot 12 + 0.7^2\cdot 12 + \cdots$

$$= 12 + \underbrace{0.7\cdot 2\cdot 12 + 0.7^2\cdot 2\cdot 12 + 0.7^3\cdot 2\cdot 12}$$

$a = 0.7\cdot 2\cdot 12$, $r = 0.7$

$$12 + \frac{0.7\cdot 2\cdot 12}{1-0.7} = \boxed{68\text{ meters}}$$

Ex 3. In a certain coutry, 55% of all income the people recieve is spent and the other 45% is saved. What is the total amount of spending in the long run generated by a stimulus of \$80 billion? (Round to 2 decimals)

$$\underbrace{\$80\text{ bill}}_{\text{original spending}} + \underbrace{0.55\cdot 80\text{ bill}}_{\text{Spent year 1}} + \underbrace{0.55^2\cdot 80\text{ bill}}_{\text{Spent year 2}} + \underbrace{0.55^3\cdot 80\text{ bill}}_{\text{Spent year 3}} + \dots$$

Notice, income for year n+1 is what is spent in year n.

$a = 80$ bill, $r = 0.55$; $\dfrac{80\text{ bill}}{1-0.55} \approx \boxed{\$177.78\text{ billion}}$

Ex 4. How much money should you invest today at an annual interest rate of 8.2% compounded continuously so that, starting 3 years from now, you can make annual withdrawals of \$3100 in perpetuity? (Round to nearest cent)

$A = Pe^{rt}$ ← continuous compounding.

$r = 0.082$, i.e., $P_t = A_t e^{-rt} = A_t e^{-0.082t}$

For each year t, want $A_t = 3100$.

$P_t = 3100e^{-0.082t}$

amount to invest for withdrawal in 3 years: $P_3 = 3100\,e^{-0.082\cdot 3}$

4 years: $P_4 = 3100\,e^{-0.082\cdot 4}$

5 years: $P_5 = 3100\,e^{-0.082\cdot 5}$

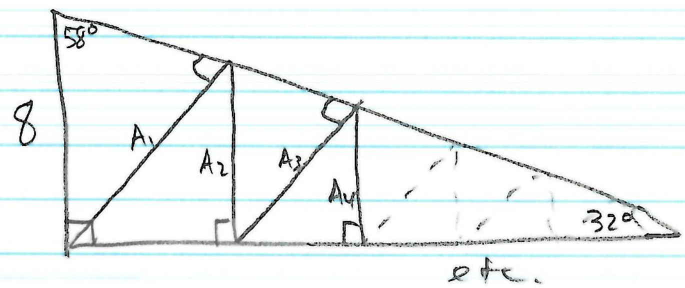
etc.

Total to invest: $3100e^{-0.082\cdot 3} + 3100e^{-0.082\cdot 4} + 3100\,e^{-0.082\cdot 5} + \dots$

$a = 3100e^{-0.082\cdot 3}$, $r = e^{-0.082}$

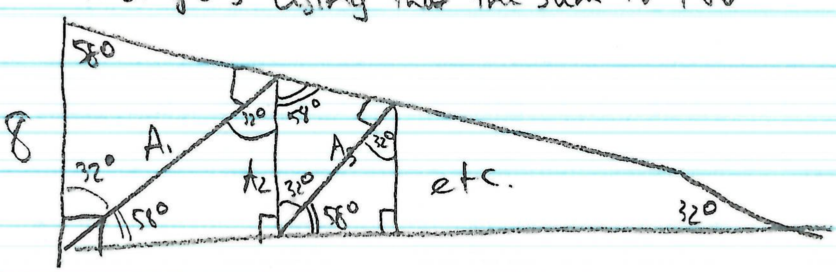
$$\frac{3100e^{-0.082\cdot 3}}{1-e^{-0.082}} \approx \boxed{\$30{,}789.02}$$

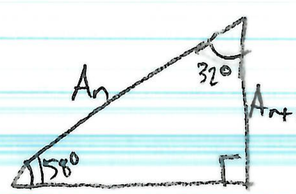
Ex 5. In a right triangle, a series of perpendicular line segments are drawn starting with an altitude using the vertex of the right angle, then subsequently drawing altitudes in the same way - from the new right angle in the triangle containg the smaller angle. Find the sum of the lengths of all of these perpendicular line segments if one angle is 58° and the side adjacent to this angle is 8 meters long.



Want $A_1 + A_2 + A_3 + A_4 + \cdots$

Notice we can find the angles for all of these triangles using that the sum is 180°



Now, for a triangle like 

By the Law of Sines, $\dfrac{A_n}{\sin(90°)} = \dfrac{A_{n+1}}{\sin(58°)}$

So $A_{n+1} = A_n \cdot \sin(58°)$

$A_1 = 8\sin(58°)$, $A_2 = 8(\sin(58°))^2$, $A_3 = 8(\sin(58°))^3, \ldots$

$a = 8\sin(58°)$, $r = \sin(58°)$; $\dfrac{8\sin(58°)}{1-\sin(58°)} \approx \boxed{44.65 \text{ m}}$