

MA 16020
Lesson 3

pg. 1

Integrals Involving Logarithms

Recall that $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$, so $\int \frac{1}{x} dx = \ln|x| + C$.
Knowing the derivative of $\ln x$ can help us with substitutions. Look for a function inside another function whose derivative is in the integral.

Ex 1. $\int 3x^{-1} \cos(\ln(2x)) dx$

Notice: $\ln(2x)$ is inside another function and its derivative ($\frac{2}{2x} = \frac{1}{x} = x^{-1}$) is in the integral.

$u = \ln(2x)$, $\frac{du}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$, so $du = \frac{1}{x} dx$

$$\begin{aligned} & \int 3 \cos(u) du \\ &= 3 \sin(u) + C \\ &= \boxed{3 \sin(\ln(2x)) + C} \end{aligned}$$

Ex 2. $\int \frac{3}{x(\ln x)^5} dx$

$\ln x$ is inside $()^5$ and $\frac{1}{x}$ is there

$u = \ln x$, $du = \frac{1}{x} dx$

$$\begin{aligned} & 3 \int u^{-5} du = 3\left(-\frac{1}{4}u^{-4}\right) + C \\ &= -\frac{3}{4}(\ln x)^{-4} + C \\ &= \boxed{-\frac{3}{(\ln x)^4} + C} \end{aligned}$$

Sometimes you will perform a substitution which will lead you to an integral like $\int \frac{1}{u} du$, which has a value of $\ln|u| + C$.

Remember that if you have a fraction like $\frac{f(x)}{g(x)}$, $g(x)$ is the "inside function" since $\frac{f(x)}{g(x)} = f(x) \cdot \frac{1}{g(x)}$ and $g(x)$ is "inside" $\frac{1}{x}$. But always look for the derivative to be present.

Ex 3. $\int \tan(2x) dx$

Notice: $\tan \theta = \frac{\sin \theta}{\cos \theta}$, so $\int \frac{\sin(2x)}{\cos(2x)} dx$

$u = \cos(2x)$, $\frac{du}{dx} = -2\sin(2x)$, $-\frac{1}{2} du = \sin(2x) dx$

$-\frac{1}{2} \int \frac{1}{u} du = -\frac{1}{2} \ln|u| + C$

$= \boxed{-\frac{1}{2} \ln|\cos(2x)| + C}$

Ex 4. $\int \frac{2t}{5+3t^2} dt$

Try $u = 5+3t^2$, $du = 6t dt$, so $\frac{1}{3} du = t dt$

$\frac{2}{6} \int \frac{1}{u} du = \frac{1}{3} \ln|u| + C = \frac{1}{3} \ln|5+3t^2| + C$

Notice: $5+3t^2$ is always positive, so $\boxed{\frac{1}{3} \ln(5+3t^2) + C}$

Ex 5. $\int \frac{1}{3x^{5/6}(1+x^{1/6})} dx$

$u = 1+x^{1/6}$, $du = \frac{1}{6} x^{-5/6} dx$, $6 du = \frac{1}{x^{5/6}} dx$

$\frac{1}{3} \int \frac{1}{(1+x^{1/6})} \frac{1}{x^{5/6}} dx = \frac{6}{3} \int \frac{1}{u} du$

$= 2 \ln|u| + C$

$= \boxed{2 \ln|1+x^{1/6}| + C}$

Reminders about application problems:

- actual change in $f(x)$ from a to b is $\int_a^b f'(x) dx$

↳ To find the value of $f(x)$ after b time units given $f'(x)$

and an initial condition, you can find

$f(x) = \int f'(x) dx + C$, use initial condition to find C

and plug in b , or "initial condition" + $\int_a^b f'(x) dx$.

- The derivative is the slope of the tangent line

- average value of $f(x)$ from a to b is

$\frac{1}{b-a} \int_a^b f(x) dx$ (using the actual function itself, not its derivative)

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Integration has Techniques, not Rules

- $\int \frac{1}{1+x} dx = \ln|1+x| + C$ (let $u=1+x$)
- $\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$ (knowing $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$)
- $\int \frac{x}{1+x} dx = 1+x - \ln|1+x| + C$ (let $u=1+x$, also $x=u-1$)
- $\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln|1+x^2| + C$ (let $u=1+x^2$)
- $\int \frac{1}{(1+x)^2} dx = -\frac{1}{1+x} + C$ (let $u=1+x$)
- $\int \frac{x}{(1+x)^2} dx = \ln|1+x| + \frac{1}{1+x} + C$ (let $u=1+x$, also $x=u-1$)

All these integrals look pretty similar, but the results can be quite different and can use different methods to find.

You actually need to think carefully to do integrals.