

MA 16020
Lesson 31
Gauss-Jordan Elimination

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Last time, we learned Gaussian elimination, which is a way to get a matrix into row-echelon form.

$$\left(\begin{bmatrix} 1 & a & b \\ 0 & 1 & c \end{bmatrix} \text{ or } \begin{bmatrix} 1 & a & b & d \\ 0 & 1 & c & e \\ 0 & 0 & 1 & f \end{bmatrix} \text{ etc.} \right.$$

diagonal entries 1, below diagonal 0)

It is very easy to read off a solution if you get the matrix into reduced row-echelon form

$$\left(\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix} \text{ etc.,} \right.$$

diagonal entries 1, below and above diagonal 0)

Gauss-Jordan Elimination

To get an augmented matrix into reduced row-echelon form

1. Perform Gaussian Elimination to get it into row echelon form.
2. Use elementary row operations to turn entries above (3,3) into 0's and then above (2,2) into 0's, etc.

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Ex 1. Transform the augmented matrix into reduced row-echelon form

$$\left[\begin{array}{ccc|c} -1 & 2 & 2 & -1 \\ 3 & 0 & 2 & -1 \\ -2 & 1 & -1 & 2 \end{array} \right]$$

$$\xrightarrow{-R_1 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 3 & 0 & 2 & -1 \\ -2 & 1 & -1 & 2 \end{array} \right] \xrightarrow{-3R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 6 & 8 & -4 \\ -2 & 1 & -1 & 2 \end{array} \right]$$

$$\xrightarrow{2R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 6 & 8 & -4 \\ 0 & -3 & -5 & 4 \end{array} \right] \xrightarrow{\frac{1}{6}R_2} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{2}{3} \\ 0 & -3 & -5 & 4 \end{array} \right]$$

$$\xrightarrow{3R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{2}{3} \\ 0 & 0 & -1 & 2 \end{array} \right] \xrightarrow{-R_3} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & \frac{4}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -2 \end{array} \right] \text{ (REF)}$$

$$\xrightarrow{-\frac{4}{3}R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right] \xrightarrow{2R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\xrightarrow{2R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$\boxed{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right]}$$

Ex 2. Use Gauss-Jordan elimination to solve the system

$$\begin{cases} 3x + y - z = -1 \\ 2x - y + 2z = 4 \\ x - 2y + z = -1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & -1 \\ 2 & -1 & 2 & 4 \\ 1 & -2 & 1 & -1 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & -1 & 2 & 4 \\ 3 & 1 & -1 & -1 \end{array} \right] \xrightarrow{-2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & 0 & 6 \\ 3 & 1 & -1 & -1 \end{array} \right]$$

$$\xrightarrow{-3R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 3 & 0 & 6 \\ 0 & 7 & -4 & 2 \end{array} \right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 7 & -4 & 2 \end{array} \right]$$

$$\xrightarrow{-7R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & -12 \end{array} \right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\xrightarrow{-R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -2 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{2R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$(0, 2, 3)$$